

ADVANCES IN PDE AND HARMONIC ANALYSIS  
A CONFERENCE IN HONOUR OF JORGE HOUNIE

Program and book of abstracts

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January 9 – 11, 2023  
São Carlos, SP, Brazil



DM / UFSCar

# Advances in PDE and Harmonic Analysis

*A conference in honour of Jorge Hounie  
on the occasion of his 75th birthday*

January 09 - 11, 2023

## Scientific Committee

Shiferaw Berhanu  
Paulo Cordaro  
Paulo Dattori da Silva  
Gustavo Hoepfner  
Tiago Picon

## Organizing Committee

Gabriel Araújo  
Camilo Campana  
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Sponsors



# Speakers

- Josefina Álvarez – New Mexico State University, USA
- Shiferaw Berhanu – University of Maryland, USA
- Paulo Cordaro – University of São Paulo, Brazil
- Galia Dafni – Concordia University, Canada
- Gustavo Hoepfner – Federal University of São Carlos, Brazil
- Ilya Kossovskiy – Masaryk University, Czech Republic
- Nicolas Lerner – Sorbonne Université, France
- Gerardo Mendoza – Temple University, USA
- Mario Milman – Instituto Argentino de Matemática, Argentina
- Tiago Picon – University of São Paulo, Brazil
- Evandro Raimundo da Silva – University of São Paulo, Brazil
- François Treves – Rutgers University, USA
- Giuliano Zugliani – State University of Campinas, Brazil

# Organization

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- Renata Oliveira Figueira – State University of Campinas, Brazil
- Alexandra Menis – Federal Technological University of Paraná, Brazil
- Luciele Rodrigues Nunes – Federal University of Rio Grande, Brazil
- Giuliano Zugliani – State University of Campinas, Brazil

# Program

## Monday, January 9

- 08:00 – 09:00 *Registration & opening*
- 09:00 – 09:50 [Álvarez](#)
- 10:00 – 10:30 *Coffee break*
- 10:30 – 11:20 [Lerner](#) (remote)
- 11:30 – 12:20 [Dafni](#) (remote)

## Tuesday, January 10

- 08:00 – 08:30 [da Silva](#)
- 08:40 – 09:00 [Ragognette](#)
- 09:10 – 09:30 [Lessa Victor](#)
- 09:40 – 10:10 *Coffee break*
- 10:10 – 11:00 [Kossovskiy](#)
- 11:10 – 11:40 [Zugliani](#)
- 12:00 – 14:00 *Lunch*
- 14:00 – 14:50 [Mendoza](#)
- 15:00 – 16:00 *Poster session*
- 16:00 – 16:50 [Treves](#) (remote)
- 19:00 *Celebration dinner*

## Wednesday, January 11

- 08:00 – 08:30 [Picon](#)
- 08:40 – 09:10 [Hoepfner](#)
- 09:20 – 09:50 *Coffee break*
- 09:50 – 10:40 [Milman](#) (remote)
- 10:50 – 11:40 [Cordaro](#)
- 11:50 – 12:00 *Picture*
- 12:00 – 12:50 [Berhanu](#) (remote)
- 13:00 *Closing*

# YouTube stream

**Monday, January 9**

<https://www.youtube.com/watch?v=NbrRTs86Ed0>

**Tuesday, January 10: morning**

<https://www.youtube.com/watch?v=MsDGUxan47M>

**Tuesday, January 10: afternoon**

<https://www.youtube.com/watch?v=uKJOfkaijNs>

**Wednesday, January 11**

<https://www.youtube.com/watch?v=OrsctmNTJ-A>

# Talks

## Josefina Álvarez

TITLE: THE DIRICHLET KERNEL IS NOT GOOD

**Abstract:** Indeed, in Shakarchi's and Stein's terminology, the Dirichlet kernel is not good. How bad it is, will be the theme of this talk, where I will prove several results, some well known and some, hopefully, not so well known, about its badness.

## Shif Berhanu

TITLE: REMARKS ON SOME OF JORGE HOUNIE'S WORKS

**Abstract:** In this talk we will present a summary of some of Jorge Hounie's research achievements with a particular focus on his joint works with Joaquim Tavares and the speaker.

## Paulo D. Cordaro

TITLE: SOME CONTRIBUTIONS OF J. HOUNIE ON THE PROBLEMS OF SOLVABILITY HYPOELLIPTICITY OF LINEAR PDE AND SYSTEMS

**Abstract:** In this talk I will present some of Hounie's results in the questions of existence and regularity of solutions to linear PDE and systems. The goal is to emphasize his contributions obtained in a remarkable academic career.

## Galia Dafni

TITLE: THE BOUNDEDNESS OF INHOMOGENEOUS CALDERÓN–ZYGmund OPERATORS ON LOCAL HARDY SPACES AND APPROXIMATE MOMENT CONDITIONS

**Abstract:** In joint work with Chun Ho Lau (Concordia), Tiago Picon (Universidade São Paulo), and Claudio Vasconcelos (Universidade Federal de São Carlos), we show the boundedness of inhomogeneous Calderón-Zygmund operators on the local Hardy spaces  $h^p$ ,  $0 < p \leq 1$ , under certain logarithmic Campanato-type cancellation conditions on the

operator. The proof is based on a new atomic and molecular decomposition using these approximate moment conditions. We also show that these conditions are necessary.

## Gustavo Hoepfner

TITLE: HARDY SPACES, FROM CLASSICAL TO MODERN

**Abstract:** In this talk I will discuss some works in which the contributions of Professor Jorge Hounie were essential, starting with my own PhD thesis to the co-supervision of an ongoing graduate research in University of Arkansas.

## Ilya Kossovskiy

TITLE: SPHERICITY AND ANALYTICITY OF A STRICTLY PSEUDOCONVEX HYPERSURFACE IN LOW REGULARITY

**Abstract:** It is well known that the sphericity of a strictly pseudoconvex real hypersurface amounts to the vanishing of its Chern-Moser tensor. The latter is computed pointwise in terms of the 6-jet of the hypersurface at a point, and thus requires regularity of the hypersurface of class at least  $C^6$ . In our joint work with Zaitsev, we apply our recent theorem on the analytic regularizability of a strictly pseudoconvex hypersurface to find a necessary and sufficient condition for the sphericity of a strictly pseudoconvex hypersurfaces of arbitrary regularity starting with  $C^3$ . Further, we obtain a simple condition for the analytic regularizability of hypersurfaces of the respective classes. Surprisingly, despite of the seemingly analytic nature of the problem, our technique is geometric and is based on the Reflection Principle in SCV.

## Nicholas Lerner

TITLE: INTEGRATING THE WIGNER DISTRIBUTION ON SUBSETS OF THE PHASE SPACE

**Abstract:** We shall review several properties of integrals of the Wigner distribution on subsets of the phase space, stressing the link between Signal Theory and Quantum Mechanics; in particular we shall use the Weyl quantization to formulate various questions related to the Wigner distribution. We will provide a theoretical proof of the invalidity of Flandrin's conjecture about integrals of the Wigner distribution on convex subsets of the phase space, a fact already proven via numerical arguments in our joint paper [\[MR4054880\]](#) with B. Delourme and T. Duyckaerts. We shall review the case of subsets of the plane whose boundary is a conic curve and show that Mehler's formula can be helpful in the analysis of these cases, including for the higher dimensional case investigated in the paper [\[MR2761287\]](#) by E. Lieb and Y. Ostrover.



## Gerardo Mendoza

TITLE: LORENTZ METRICS WITH SINGULARITIES

**Abstract:** I'll discuss a class of Lorentz metrics on manifolds with boundary that are singular on the boundary. The light cone of these metrics collapses to a line bundle over the boundary, while making the latter be space-like with total volume 0. I'll present necessary conditions for the existence of such metrics on a compact manifold, and describe some interesting properties of the bicharacteristic flow of their D'Alembertian. For these I'll pose a Cauchy problem and, time permitting, describe the Fourier Integral Operator associated as solution operator near the boundary. This is joint work with Zachary Hanson-Hart.

## Mario Milman

TITLE: YUDOVICH AND VISHIK THEORIES REVISITED: EXTRAPOLATION METHODS

**Abstract:** We revisit the classical methods developed by Yudovich and Vishik to establish uniqueness of solutions for the 2D Euler equations for an inviscid incompressible fluid on  $\mathbb{T}$  or  $\mathbb{R}^2$ . We show that extrapolation theory provides a framework that unifies both approaches and allows for meaningful extensions out of the reach of the classical results.

Joint work with Oscar Dominguez (Universidad Complutense de Madrid).

## Tiago Picon

TITLE:  $L^\infty$  SOLVABILITY OF ELLIPTIC AND CANCELING HOMOGENEOUS LINEAR EQUATIONS ON MEASURES

**Abstract:** In this talk, we present new results on  $L^\infty$  solvability of the equation  $A^*(D)f = \mu$ , where  $A(D)$  is an elliptic and canceling homogeneous differential operator and  $\mu$  is a nonnegative measure. Our method is based on type  $L^1$  estimates on measures for a special class of vector fields.

This is joint work with Victor Biliatto (UFSCar).

## Evandro Raimundo da Silva

TITLE: LOCAL SOLVABILITY FOR A CLASS OF LINEAR OPERATORS IN BESOV AND TRIEBEL-LIZORKIN SPACES

**Abstract:** We show local solvability in Besov and Triebel-Lizorkin spaces for a class of first order linear operators  $L$  defined on an open set of  $\mathbb{R}^{n+1}$ ,  $n \in \mathbb{N}$ , satisfying the condition  $(P)$  of Nirenberg-Treves and whose coefficients are Hölder continuous.

## François Trèves

TITLE: MICROLOCAL REPRESENTATION OF SOLUTIONS OF ANALYTIC PSEUDODIFFERENTIAL EQUATIONS OF PRINCIPAL TYPE

**Abstract:** The lecture outlines the proof of the distribution version of J.-M. Trépreau's theorem on the microlocal solvability of an analytic pseudodifferential equation  $Pu = f$  in *hyperfunctions*. The main theorem in the lecture is not a consequence of Trépreau's theorem and the proof follows a radically different approach. The necessary condition **(Psi)** allows us to construct, in a conic neighborhood of a point  $\varphi \in \mathbb{R}^N \times (\mathbb{R}^N \setminus \{0\})$ , a Fourier Integral Operator  $G$  with complex phase-function  $\varphi$ , an *FBI phase-function* in the sense of Sjöstrand, such that  $PGf - f$  is microanalytic at  $\varphi$ , i.e.  $\varphi$  does not belong to the analytic wave-front set of  $PGf - f$ . Condition **(Psi)** is reflected in the behavior of  $\text{Im}\varphi$ . The technical details of the proofs of the successive steps can be found in the late chapters of the recent book of the author in the Springer's *Grundlehren series*.

## Giuliano Zugliani

TITLE: GLOBAL HYPOELLIPTICITY OF COMPLEX VECTOR FIELDS ON NONCOMPACT SURFACES

**Abstract:** We consider smooth, non-vanishing complex vector fields (not essentially real) that satisfy the Nirenberg-Treves condition. We give a characterization of a globally hypoelliptic vector field  $L$  in terms of the topological type of the equivalence classes determined by a standard equivalence relation defined on the set of non elliptic points of  $L$ . Some of the consequences are that globally hypoelliptic vector fields are globally solvable and that if a surface carries a globally hypoelliptic vector field, it must be parallelizable. We also present some examples of globally hypoelliptic vector fields.

This is a joint work with Jorge G. Hounie from UFSCar. We thank FAPESP and CNPq for the financial support.

# Short talks

## Bruno de Lessa Victor (USP)

TITLE: GLOBAL ANALYTIC HYPOELLIPTICITY OF INVOLUTIVE SYSTEMS ON COMPACT MANIFOLDS

**Abstract:** Given  $M$  a compact, connected and orientable, real-analytic manifold, and closed, real-valued, real-analytic 1-forms  $\omega_1, \dots, \omega_m$  on  $M$ , we characterize the global analytic hypoellipticity of the first operator featuring in the differential complex over  $M \times \mathbb{T}^m$  naturally associated to a involutive system of vector fields determined by them. Global Gevrey hypoellipticity is determined simultaneously.

## Bibliography

- [1] G. ARAÚJO, P. L. DATTORI DA SILVA, B. DE LESSA VICTOR. *Global analytic hypoellipticity of involutive systems on compact manifolds*. *Mathematische Annalen*, 1-26 (2022).

## Luis Fernando Ragognette (UFMG)

TITLE: GLOBAL ANALYTIC HYPOELLIPTICITY AND SOLVABILITY OF CERTAIN OPERATORS SUBJECT TO GROUP ACTIONS

**Abstract:** Consider the compact real-analytic manifold  $M$  and  $P$  a differential operator with real-analytic coefficients on  $M$ . In a joint work with Gabriel Araujo and Igor Ferra, we investigated necessary conditions for global solvability of  $P$  in the analytic setup.

Moreover, when  $M = T \times G$ , where  $T$  is a compact analytic manifold and  $G$  is a compact Lie group, we considered a class of differential operators on  $M$  that are elliptic in  $T$  and invariant by left translations on  $G$ . We were able to prove for operators in this class that global hypoellipticity implies global analytic-hypoellipticity.

# Poster session

## Victor Sandrin Biliatto (UFSCar)

TITLE: LEBESGUE SOLVABILITY OF ELLIPTIC HOMOGENEOUS LINEAR EQUATIONS WITH MEASURE DATA

**Abstract:** In this work we present new results on solvability of the equation  $A^*(D)f = \mu$  for  $f \in L^p(\mathbb{R}^n)$  and positive measure data  $\mu$  associated to an elliptic homogeneous differential operator  $A(D)$  of order  $m$ . Our method is based on controlling the  $(m, p)$ -energy of  $\mu$  giving a natural characterization for solutions when  $1 \leq p < \infty$ . We also obtain sufficient conditions in the limiting case  $p = \infty$  using new  $L^1$  estimates on measures for elliptic and canceling operators.

This is a joint work with Tiago Picon (FFCLRP-USP).

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- [1] BILIATTO, V. S. & PICON, T. H., *A note on Lebesgue solvability of elliptic homogeneous linear equations on measures*, arXiv:2210.09768 [math.AP].
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- [7] VAN SCHAFTINGEN, J., *Limiting Sobolev inequalities for vector fields and canceling linear differential operators*, J. Eur. Math. Soc., **5**, 3 (2013), 877-921.

## Joel Coacalle (USP)

TITLE: SUBELLIPTIC ESTIMATES FOR THE COMPLEX GREEN OPERATOR ON NON PSEUDOCONVEX CR MANIFOLDS OF HYPERSURFACE TYPE

**Abstract:** In this work we obtain subelliptic estimates for the complex Green operator  $K_q$  at some fixed level of  $(0, q)$ -forms of the  $\bar{\partial}_b$ -complex associated to a non pseudoconvex CR manifolds of hypersurface type  $M$ . The nonpseudoconvex CR manifolds we consider are the weak  $Y(q)$  manifolds, and there we assume a finite commutator type property. We introduce a condition that compares sum of  $q$  eigenvalues of the Levi matrix, weaker than the well known  $D(q)$  condition, adapted to weak  $Y(q)$  case. We show that these two conditions, finite type and the eigenvalue sum comparison conditions are sufficient to show subelliptic estimates for  $K_q$  at some specific level of  $(0, q)$ -forms using a microlocal argument.

## Stefan Fördös (USP)

TITLE: ELLIPTICITY AND THE PROBLEM OF ITERATES IN ULTRADIFFERENTIABLE CLASSES

**Abstract:** In 1978 Métivier showed that a differential operator  $P$  with analytic coefficients is elliptic if and only if the theorem of iterates holds for  $P$  with respect to any non-analytic Gevrey class. We extend this theorem to Denjoy-Carleman classes given by strongly non-quasianalytic weight sequences. Moreover, we point out that the analogous statement for Braun-Meise-Taylor classes given by weight functions cannot hold. This signifies an important difference in the properties of Denjoy-Carleman classes and Braun-Meise-Taylor classes, respectively.

This is joint work with Gerhard Schindl from the University of Vienna.

## Renan Dantas Medrado (UFAL)

TITLE: AN EXTENDED CLASS OF FBI TRANSFORM WITH APPLICATIONS TO ULTRADIFFERENTIAL REGULARITY

**Abstract:** We introduce a class of FBI transforms using weight functions (which includes the class of FBI transformations used by M. Christ in the Gevrey regularity study) that is well suited when dealing with ultradifferentiable functions and ultradistributions defined by weight functions in the sense of Braun, Meise and Taylor (BMT). We show how to characterize local regularity of BMT ultradistributions using this wider class of FBI transform and, as an application, we characterize the BMT vectors and prove a relation between BMT local regularity and BMT vectors.

## Luís Márcio Salge (IFMG)

TITLE: SPECTRUM OF DIFFERENTIAL OPERATORS WITH ELLIPTIC ADJOINT ON A SCALE OF LOCALIZED SOBOLEV SPACES

**Abstract:** In this work we present a complete study about the spectrum of a constant coefficients differential operator of order  $m \in \mathbb{N}$ ,  $a(D)$ , whose adjoint  $a(D)^*$  is elliptic, seen as a pseudo-differential operator on a interval  $I \subset \mathbb{R}$ , that is, seen as  $a(D) : H_0^{s+m}(I) \subset H_{loc}^s(I) \rightarrow H_{loc}^s(I)$ ,  $s \in \mathbb{R}$ . Here,  $H_{loc}^s(I)$  is endowed with the topology generated by a family of seminorms  $(p_j^{(s)})_{j \in \mathbb{N}}$  given by  $p_j^{(s)}(f) \doteq \|\varphi_j f\|_{H^s(\mathbb{R})}$ ,  $f \in H_{loc}^s(I)$ , where, for each  $j \in \mathbb{N}$ ,  $I_j = (a_j, b_j)$  is such that  $[a_j, b_j] \subset (a_{j+1}, b_{j+1})$ , with  $I = \bigcup_{j \in \mathbb{N}} [a_j, b_j]$ , and  $\varphi_j \in C_c^\infty(I_{j+1})$  satisfies  $\varphi_j = 1$  in  $[a_j, b_j]$ .

When we indicate  $a(D)$  as above, we mean that in  $H_0^{s+m}(I)$  we consider the topology induced from  $H_{loc}^s(I)$ .

This study was developed inspired by what happen with the Laplace operator on  $L^2(I)$ . Here, we replace  $L^2(I)$  by  $H_{loc}^s(I)$  and  $H_0^1(I) \cap H^2(I)$  by  $H_0^{m+s}(I)$ , as suggested by the definitions by Dan Henry for pseudodifferential operators.

The best conclusions we obtain are when we consider the Laplace operator on an interval  $I$  as  $\Delta : H_0^2(I) \subset L_{loc}^2(I) \rightarrow L_{loc}^2(I)$ . For it, we calculate its closure and compare its spectrum in three stages:

- (1) When it is defined on  $H_0^2(I)$ .
- (2) When its domain is  $H_0^1(I) \cap H^2(I)$ , where we call it  $\Delta_{L^2}$ ; and
- (3) When it is defined on  $H_{loc}^2(I)$ . This, as we are going to see, is the domain of the closure  $\overline{\Delta}$ .

In particular, we prove that  $\sigma_c(\Delta) = \sigma_r(\Delta^*) = \sigma_p(\overline{\Delta}) = \mathbb{C}$  and  $\sigma_c(\Delta_{L^2}) = \mathbb{C} \setminus \left\{ -\frac{\pi^2 n^2}{l(I)^2} : n \in \mathbb{N} \right\}$ , where  $l(I)$  is the length of  $I$ .

Joint work with Éder Rítis Aragão Costa (ICMC-USP).

## Gabriel Mendes do Valle (UNICAMP)

TITLE: WEAK\* TOPOLOGY AND THE BANACH-ALAOGLU THEOREM

**Abstract:** We will discuss some aspects of the weak\* topology and some of its uses when studying linear operators. We also want to present a proof of the Banach-Alaoglu theorem and show how this important result relates to properties of normed vector spaces such as reflexivity and separability.

We thank CNPq for the financial support (via PICME).

## Claudio Vasconcelos (UFSCar)

TITLE: NECESSARY AND SUFFICIENT CONDITIONS FOR THE CONTINUITY OF INHOMOGENEOUS CALDERÓN–ZYGmund TYPE OPERATORS IN  $h^p(\mathbb{R}^n)$

**Abstract:** In this work we present a new approach to atoms and molecules on Goldberg’s local Hardy spaces  $h^p(\mathbb{R}^n)$  for  $0 < p \leq 1$ , replacing the local vanishing moment condition by an appropriate control of its size. As applications, we show improved necessary and sufficient conditions for the continuity of inhomogeneous Calderón-Zygmund operators and its strongly singular version.

Joint work with Galia Dafni, Chun Ho Lau (Concordia University) and Tiago Picon (Universidade de São Paulo).

### Bibliography

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## Miguel Dario Soto Vieira (UFRJ)

TITLE: NULL-CONTROLLABILITY PROPERTY FOR THE KAWAHARA EQUATION

**Abstract:** We study the controllability properties of the linear Kawahara equation posed on a periodic domain. We show that the equation is null-controllable by means of a control depending only on time and acting on the system through a given shape function in space. The method we apply is based on Fourier expansion of solutions and the analysis of a biorthogonal sequence to a family of complex exponential functions.

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