# Conference on Linear PDEs
## celebrating José Ruidival do Santos Filhos career

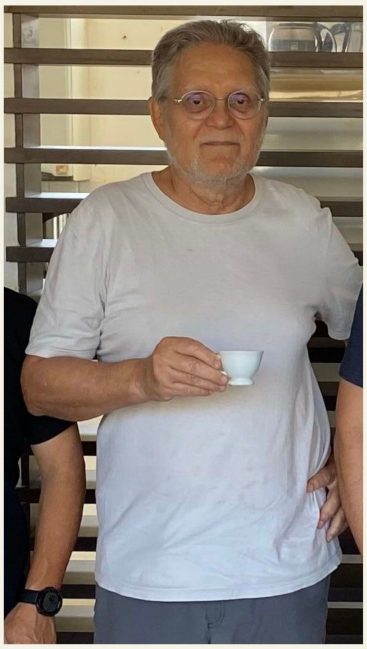
# Program and book of abstracts

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February 1 – 2, 2024
São Carlos, SP, Brazil

# Conference on Linear PDE's

*celebrating José Ruidival do Santos Filho's career*



**February, 01 and 02, 2024.**
**São Carlos - Brazil**




**Scientific committee**

**Paulo Cordaro – University of São Paulo.**
**Marcelo Ebert – University of São Paulo.**
**Gustavo Hoepfner – Federal University of São Carlos**
**Jorge Hounie – Federal University of São Carlos.**
**Irina Mitrea – Temple University.**

**Organizing committee**

**Max Reinhold Jahnke – Universität zu Köln.**
**Rafael Kapp – Federal University of São Carlos.**
**Luciele Rodrigues Nunes, Federal University of Rio Grande.**
**Tiago Henrique Picon – University of São Paulo.**

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**Sponsors**

**Organization**

# Speakers

- Gabriel Araújo, University of São Paulo, Brazil
- Francisco Braun, Federal University of São Carlos, Brazil
- Marcelo Ebert, University of São Paulo, Brazil
- Cezar Issao Kondo, Federal University of São Carlos, Brazil
- Renan Medrado, Federal University of Alagoas, Brazil
- Gerardo Mendoza – Temple University, USA
- Irina Mitrea, Temple University, USA
- Pedro Takemura, Baylor University, USA
- Marco Antonio Teixeira, University of Campinas, Brazil
- Severino Toscano, University of São Paulo, Brazil
- Dmitri Zaitsev, Trinity College Dublin, Ireland

# Organization

## Scientific committee

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- Irina Mitrea – Temple University, USA

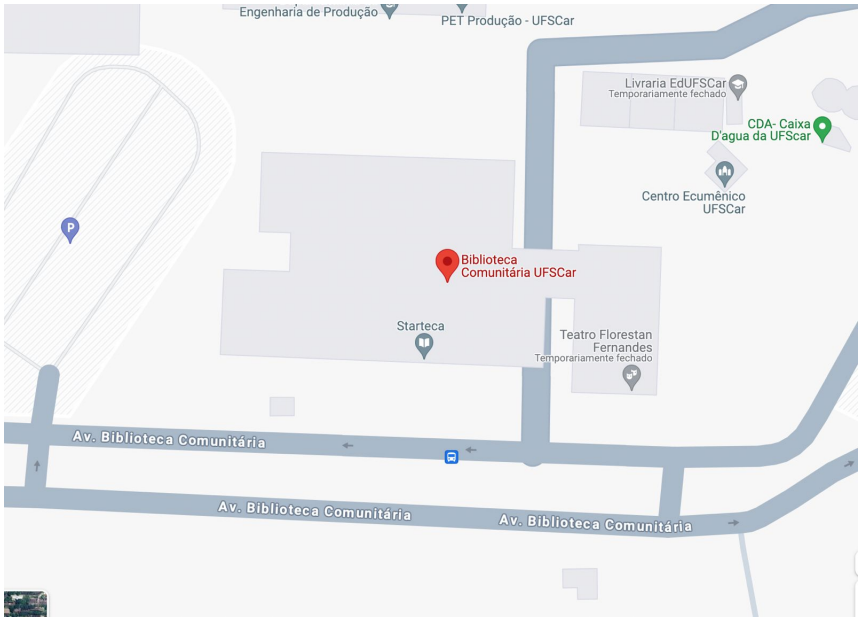
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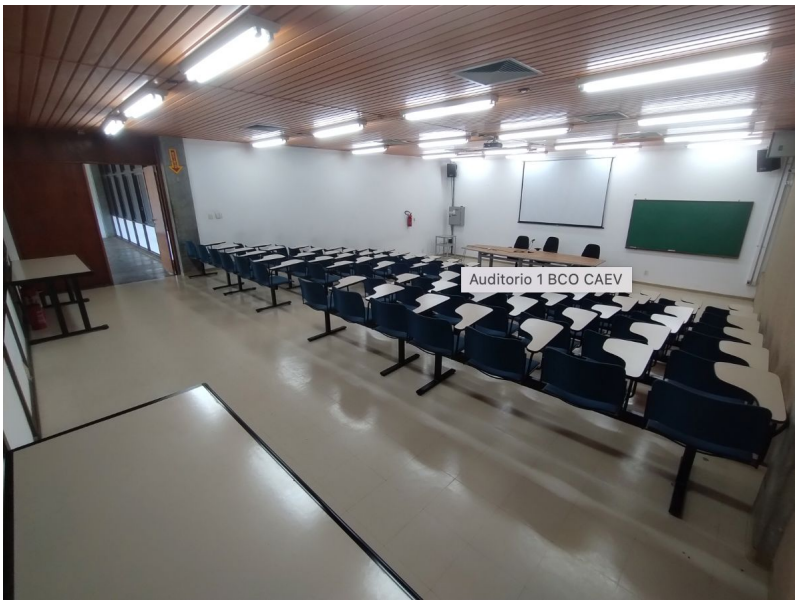
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- Rafael Augusto dos Santos Kapp – Federal University of São Carlos, Brazil
- Luciele Rodrigues Nunes – Federal University of Rio Grande, Brazil
- Tiago Henrique Picon – University of São Paulo, Brazil

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4 Centro de Convivência Sul | Athenas Paulista (Venda de passes)
5 Departamento de Assistência Médica e Odontológica (DeAMO) | Departamento de Serviço Social (DeSS)
6 Divisão de Controle Acadêmico (DICA)
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14 Pavilhão de Ginástica | Ginasinho
15 Rádio UFSCar
16 Restaurante Universitário (RU)
17 Salas de Aula (AT1) | Babilônia I
18 Salas de Aula (AT2) | Babilônia II | Auditório do CECH
19 Salas de Aula (AT4)
20 Salas de Aula (AT5)
21 Salas de Aula (AT6)
22 Salas de Aula (AT7)
23 Salas de Aula (AT8)
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8 Departamento de Matemática (DM)
9 Departamento de Química (DQ)
10 Depósito de Produtos Químicos do DQ
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13 Laboratório de Caracterização Estrutural (LCE) do DEMa
14 Laboratório de Destilação do DQ
15 Laboratório Interdisciplinar de Eletroquímica e Cerâmica (LIEC) do DQ
16 Laboratório de Engenharia do Produto do DEP
17 Laboratório de Ensino de Química do DQ
18 Laboratório de Física e Química do CCET
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17 Laboratório do DB
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19 Laboratório de Neuropediatria
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24 Revista Brasileira de Fisioterapia
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Praça da Bandeira
Vegetação
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**Address:** Auditório I - BCO



# Program

## Thursday, February 1

| | |
|---|---|
| 08:00 – 09:00 | *Registration & Opening* |
| 09:00 – 09:40 | Irina Mitrea |
| 09:50 – 10:30 | Dmitri Zaitsev |
| 10:40 – 11:10 | *Coffee break* |
| 11:10 – 11:50 | Marcelo Ebert |
| 12:00 – 14:00 | *Lunch* |
| 14:00 – 14:40 | Gerardo Mendoza |
| 14:50 – 15:30 | Gabriel Araújo |
| 15:40 – 16:30 | *Coffee break* and *Poster Session* |
| 16:30 – 17:10 | Pedro Takemura |
| 19:00 | *Celebration dinner* |

## Friday, February 2

| | |
|---|---|
| 09:00 – 09:40 | Severino Toscano |
| 09:50 – 10:30 | Cezar Kondo (in portuguese) |
| 10:40 – 11:10 | *Coffee break* |
| 11:10 – 11:50 | Renan Medrado |
| 12:00 – 12:10 | *Photo* |
| 12:00 – 14:00 | *Lunch* |
| 14:00 – 14:40 | Marco Antonio (in portuguese) |
| 14:50 – 15:30 | Francisco Braun |
| 15:40 – 16:10 | *Coffee break* |
| 16:10 – 16:30 | Closing |

# Talks

## Gabriel Araújo - USP

**Title:** Tube structures with extra symmetries.

**Abstract:** I'll discuss recent results concerning global solvability, normalization and representation of the cohomology spaces of corank $m$ real tube structures on $M \times \mathbb{T}^m$, where $M$ is a compact Lie group, extending previous results when $M$ is an $n$-torus (Bergamasco-Petronilho, 1999), (Dattori-Meziani, 2016).

Joint work with Igor A. Ferra (UFABC), Max R. Jahnke (Köln) and Luis F. Ragognette (UFMG). Supported by FAPESP and CNPq - INCTMat.

## Francisco Braun - UFSCar

**Title:** The global invertibility problem: some results in the polynomial case.

**Abstract:** The problem of finding conditions guaranteeing a local invertible map is globally invertible goes back at least to the works of Hadamard, in the early 19th century. Although important advances were made since then, yielding beautiful results in different areas of Mathematics, several open problems still remain unsolved. Maybe the most known open problem here is the Jacobian conjecture, raised by Keller in 1939, claiming that a polynomial map $F : \mathbb{K}^n \to \mathbb{K}^n$, where $\mathbb{K}$ is a field of characteristic 0 and $n \geq 2$, becomes an automorphism provided its Jacobian determinant is a nonzero constant. If $\mathbb{K} = \mathbb{R}$, the *strong* claim that $F$ is invertible if its Jacobian determinant is different from zero throughout $\mathbb{R}^n$, meaning $F$ is locally invertible, is disproved by a celebrated counterexample provided by Pinchuk in 1994.

In this lecture, I will discuss the polynomial case in $\mathbb{R}^n$, presenting conditions for the global invertibility of $F$ as well as counterexamples to certain versions of the Jacobian conjecture. My introduction to this subject occurred around 2010, through Prof. Ruidival, during my Ph.D. under his supervision. Congratulations, Prof. Ruidival!

## Marcelo Rempel Ebert - USP

**Title:** Sharp $L^p - L^q$ estimates for a class of dissipative wave equations.

**Abstract:** In this talk we derive $L^p - L^q$ estimates, with $1 \leq p \leq q \leq \infty$ (including endpoint estimates as $L^1 - L^1$ and $L^1 - L^\infty$), for the solution to

$$u_{tt} - \Delta u + Au_t = 0, \quad t \in \mathbf{R}_+,\, x \in \mathbf{R}^n,$$
$$u(0,x) = 0, \quad u_t(0,x) = u_1(x),$$

for a general class of dissipation terms, where $Af = \mathcal{F}^{-1}(a(\xi)\,\mathcal{F}f(\xi))$, with $a \in \mathcal{C}^{n+1}(\mathbf{R}^n \setminus \{0\}) \cap \mathcal{S}'(\mathbf{R}^n)$, and $a(\xi) > 0$ verifies conditions of Mikhlin-Hörmander type for $M^q_p$ multipliers which may be different at low frequencies and at high frequencies; in particular $a(\xi)$ may also be inhomogeneous and anisotropic. We prove that the obtained estimates are sharp.

This talk is a joint work with Marcello D'Abbicco, University of Bari.

## Cezar Kondo - UFSCar

**Title:** Alguns Resultados de Unicidade para Algumas Equações.

**Abstrac:** Nesta palestra, iremos falar de alguns resultados de unicidade de algumas equações como equações do tipo Kawahara e dentre outras obtidos em colaboração com Ruidival dos Santos Filho, Marcos de Farias e Miguel Zapata. Discutiremos a técnica aplicada nestas equações, que foi introduzida por C. E. Kenig, G. Ponce e L. Vega para equações do tipo KdV, a sua limitação e, consequentemente em quais tipos de equações que ela pode ser aplicada.

## Renan Medrado - UFAL

**Title:** Local and microlocal BMT regularity via a class of FBI transforms.

**Abstract:** In this talk I will present a characterization of local and microlocal BMT regularity (in the sense of Braun, Meise and Taylor) for ultradistributions using a class of FBI transforms defined by weight functions (this class involves the weight functions used to define BMT regularity and includes the subclass of Sjostrands FBI transforms used by M. Christ in 1997). Also, I will present a characterization of Iterates of Partial Differential Operators in the sense of BMT classes (called BMT vectors) and, as an application, I will present a relation between BMT regularity and BMT vectors. Joint work with G. Hoepfner. Partially supported by FAPEAL.

## Gerardo Mendoza - Temple University

**Title:** The Friedrichs extension of semibounded elliptic second order wedge operators.

**Abstract:** A compact manifold with fibered boundary is a manifold $\mathcal{M}$ whose boundary $\mathcal{N}$ is the total space of a locally trivial fibration $\wp : \mathcal{N} \to \mathcal{Y}$ with typical fiber $\mathcal{F}$. Associated to such manifolds there is a class of differential operators which near any point of the boundary have the form $A = x^{-m}P$ where $P$ has the representation

$$P = \sum_{|\alpha|+|\beta|+k} a_{k\alpha\beta}(xD_x)^k(xD_y)^\alpha D_z^\beta$$

with smooth coefficients $a_{k\alpha\beta}$ in coordinates $(x, y, z)$ where $x$ is a defining function for $\mathcal{N}$, the sets $\{(x, y, z) : x = 0,\ y = \text{constant}\}$ are the fibers of $\wp$. Such $P$ is called an edge operator (R. Mazzeo), and $A$ (B.-W. Schulze) is a wedge operator. There is a natural notion of ellipticity for $A$. I will discuss the Friedrich extension of a second order ($m = 2$) $w$-elliptic operator $A$ acting on sections of a Hermitian vector bundle under a mild assumption on its boundary spectrum. This is joint work with T. Krainer.

## Irina Mitrea - Temple University

**Title:** Generalized Banach Function Spaces, Extrapolation, and Singular Integrals.

**Abstract:** Generalized Banach Function Spaces, Extrapolation, and Singular Integrals Abstract: This talk, at the interface between Harmonic Analysis, Partial Differential Equations, Functional Analysis, and Geometric Measure Theory, focuses on the use of extrapolation techniques for solving boundary value problems via singular integral methods in irregular domains. This is joint work with Dorina Mitrea and Marius Mitrea from Baylor University and is part of our five volume Geometric Harmonic Analysis Series.

## Pedro Takemura - Baylor University

**Title:** The Dirichlet Problem on Grand Herz Spaces.

**Abstract:** In this talk we introduce a new brand of Herz spaces, which we refer to as Grand Herz spaces, and this is accomplished through the lens of Generalized Banach Function Spaces (GBFS). Remarkably, we find that Grand Herz spaces themselves fall under the scope of GBFS, thus opening the door for the treatment of boundary value problems in such a functional analytic setting. Specifically, we present a well-posedness result for the Dirichlet Problem on rough domains with boundary data prescribed in Grand Herz spaces. This is joint work with Marius Mitrea.

## Marco Antonio Teixeira - UNICAMP

**Title:** A Walk between Refractive Systems.

**Abstract:** In this talk we discuss some qualitative facets of a class of non-smooth dynamical systems (NSDS) in dimension 3: Refractive Systems. First of all, we present a classification of generic singularities. Some global phenomena, are also discussed. It is worth to say that, we deal with discontinuous vector fields in $\mathbb{R}^3$ where the discontinuities are concentrated on a 2-dimensional surface.

## Severino Toscano Melo - USP

**Title:** An abstract characterization of pseudodifferential operators conjectured by Marc Rieffel.

**Abstract:** Two decades ago, Marcela Merklen and I obtained partial results on a Rieffel conjecture that would characterize a class of pseudodifferential operators with symbols taking values in a C*algebra A as the operators with C-infinity orbit for a certain action of the Heisenberg group on the adjointable operators of a Hilbert module (a generalization of the space $L^2(\mathbb{R}^n)$). That construction appears in the context of "strict deformation quantization", one of many possible approaches to provide a rigorous basis to the principle of correspondence in Quantum Mechanics.

In this talk, I intend to report on recent work in collaboration with Rodrigo Cabral that completely solves the Rieffel conjecture in the case where A has unity, using new techniques.

## Dmitri Zaitsev - Trinity College Dublin

**Title:** Catlin's global regularity – a new approach and generalizations.

**Abstract:** The celebrated result of Catlin on global regularity of the $\bar{\partial}$-Neumann operator for pseudoconvex domains of finite type links local algebraic- and analytic geometric invariants through potential theory with estimates for $\bar{\partial}$-equation. Yet despite their importance, there seems to be a major lack of understanding of Catlin's techniques, resulting in a notable absence of an alternative proof, exposition or simplification.

The goal of my talk will be to present an alternative proof based on a new notion of a "tower multi-type". The finiteness of the tower multi-type is an intrinsic geometric condition that is more general than the finiteness of the regular type, which in turn is more general than the finite type. Under that condition, we obtain a generalized stratification of the boundary into countably many level sets of the tower multi-type, each covered locally by strongly pseudoconvex submanifolds of the boundary. The existence of such stratification implies Catlin's celebrated potential-theoretic "Property (P)", which, in

turn, is known to imply global regularity via compactness estimate. Notable applications of global regularity include Condition R by Bell and Ligocka and its applications to boundary smoothness of proper holomorphic maps generalizing a celebrated theorem by Fefferman.

# Poster session

## Artur H. de Oliveira Andrade - Temple University

**Title:** Overdetermined Boundary Value Problems for the Stokes System in Uniformly Rectifiable Domains.

**Abstract:** The subject of this poster is the analysis of Overdetermined Boundary Value Problems (OBVP) for the Stokes system of linearized hydrostatics in in uniformly rectifiable domains with boundary datum in $L^p$ based function spaces with integrability exponent in the interval $(1, \infty)$. This analysis includes integral representation formulas, jump formulas, existence and uniqueness of solutions for the OBVP, and classical Hardy spaces associated with the Stokes system.

This is joint work with Irina Mitrea (Temple University), Dorina Mitrea and Marius Mitrea (Baylor University).

## Victor S. Biliatto - UFSCar

**Title:** Hausdorff dimension of removable sets for elliptic and canceling homogeneous differential operators.

**Abstract:** In this work we give an upper bound on the Hausdorff dimension of removable sets for elliptic and canceling homogeneous differential operators with constant coefficients in the class of bounded functions, using a simple extension of Frostman's lemma in Euclidean space with an additional power decay.

This is a joint work with Laurent Moonens (Université Paris-Saclay) and Tiago Picon (Universidade de São Paulo).

### Bibliography

[1] Biliatto, V., Moonens, L. and Picon, T., *Hausdorff dimension of removable sets for elliptic and canceling homogeneous differential operators in the class of bounded functions*, Preprint, `https://doi.org/10.48550/arXiv.2312.02560`

[2] Biliatto, V. and Picon, T., *A note on Lebesgue solvability of elliptic homogeneous linear equations with measure data*, J. Geom. Anal., v. 34, 22 (2024)

[3] Harvey, R. and Polking, J., *Removable singularities of solutions of linear partial differential equations*, Acta Math., v. 125 (1970)

[4] Mattila, P., *Geometry of sets and measures in Euclidean spaces - Fractals and rectifiability*, Cambridge University Press, Cambridge (1995) [Cambridge Studies in Advanced Mathematics, vol. 44]

[5] Moonens, L., *Removable singularities for the equation* $\operatorname{div} v = 0$, Real Anal. Exchange (2006)

[6] Phuc, N. C. and Torres, M., *Characterizations of the existence and removable singularities of divergence-measure vector fields*, Indiana Univ. Math. J., v. 57, n. 4 (2008)

# Joel Coacalle - USP

**Title:** New approach of local Hardy spaces on homogeneous type spaces.

**Abstract:** Let $(X, d, \mu)$ be a space of homogeneous type. In this work, we establish a new atomic and molecular theory of local Hardy space $h^p(X)$ for $0 < p \leq 1$. The idea is inspired in the recent work due to G. Dafni et al. [2] introducing a nonhomogeneous cancellation condition of log-type. Several properties including and a new characterization of the dual space is presented.

This is joint work with Tiago Picon (University of São Paulo) and Claudio Vasconcelos (Université Paris-Saclay). The author acknowledges support from FAPESP, grant number 2022/02211-8.

## Bibliography

[1] Ronald R Coifman and Guido Weiss. Extensions of Hardy spaces and their use in analysis. *Bulletin of the American Mathematical Society*, 83(4):569–645, 1977.

[2] Galia Dafni, Chun Ho Lau, Tiago Picon, and Claudio Vasconcelos. Inhomogeneous cancellation conditions and Calderón–Zygmund type operators on $h^p$. *Nonlinear Analysis*, 225:113110, 2022.

[3] Ziyi He, Dachun Yang, and Wen Yuan. Real-variable characterizations of local Hardy spaces on spaces of homogeneous type. *Mathematische Nachrichten*, 294(5):900–955, 2021.

# Jeongsu Kyeong - Temple University

**Title:** The poly-Cauchy operator in uniformly rectifiable domains.

**Abstract:** The classical Cauchy integral operator is one of the most famous and most studied singular integral operator in mathematics. In this talk, I will be presenting a higher-order analogue of the existing theory for the classical Cauchy operator, in which the salient role of the Cauchy-Riemann operator $\overline{\partial}$ is now played by natural powers of this. A central role will be played by integral representation formulas, jump relations and higher-order Fatou-type theorems.

This is joint work with Irina Mitrea (Temple University), Dorina Mitrea and Marius Mitrea (Baylor University).

# Antonio Lagioia - University of Bari

**Title:** Critical curves for a weakly coupled system of $\sigma$-evolution equations with noneffective damping.

**Abstract:** We will consider the weakly coupled system

$$\begin{cases} \partial_t^2 U + (-\Delta)^\sigma U + D\partial_t U = F(U), & t > 0,\ x \in \mathbb{R}^n, \\ (U, \partial_t U)(0, x) = (0, \Phi(x)), & x \in \mathbb{R}^n, \end{cases}$$

where $\sigma > 1$, $U = (u_1, u_2)^\intercal$, $\Phi = (\varphi_1, \varphi_2)^\intercal$,

$$DU = \begin{pmatrix} a_1(-\Delta)^{\frac{\theta_1}{2}} & 0 \\ 0 & a_2(-\Delta)^{\frac{\theta_2}{2}} \end{pmatrix}, \quad \theta_1, \theta_2 \in [\sigma, 2\sigma],\ a_1, a_2 \geq 0,$$

is a damping term in the noneffective case, and $F(U) = (f_1(U), f_2(U))^\intercal$ is a semilinear coupling, for istance

$$F(U) = \begin{pmatrix} c_{11}|u_1|^{p_{11}} + c_{12}|u_2|^{p_{12}} \\ c_{21}|u_1|^{p_{21}} + c_{21}|u_2|^{p_{22}} \end{pmatrix},$$

We will show how the $L^p - L^q$ estimates for the corresponding linear equation, obtained in [1], are crucial for determining the so-called critical curves for the existence of small data solutions of the problem and for discussing the space dimension of existence validity.

## Bibliography


[1] M. D'Abbicco, M.R. Ebert, *$L^p - L^q$ estimates for a parameter-dependent multiplier with oscillatory and diffusive components*, Journal of Mathematical Analysis and Applications **504** (2021).

[2] M. D'Abbicco, A. Lagioia, *A new critical curve for systems of evolution equations with semilinear coupling*, preprint.

## Catarina Barbosa Machado - UFSCar

**Title:** Nonhomogeneous local div-curl type estimates for elliptic linear differential operators.

**Abstract:** We extend the classical div-curl inequality proved in [1] to the local setup of higher order elliptic linear differential operators $A(x, D)$ with smooth coefficients in localizable Hardy spaces $h^p$ for $0 < p < 1$. The tools include a generalized Poincaré-type inequality and an appropriate atomic decomposition on localizable Sobolev-Hardy spaces recovering the previous results for first order operators in [4]. The end point case for $p = 1$ is presented in the setting of elliptic system of complex vector fields inspired in the works [2, 3]. This is a joint work with Tiago Picon (University of São Paulo).

### Bibliography

[1] R. Coifman, P. Lions, Y. Meyer and S. Semmes, *Compensated compactness and Hardy spaces*, J. Math. Pures Appl., Vol. 72 (1993), 247-286.

[2] G. Dafni, *Nonhomogeneous div-curl lemmas and local Hardy spaces*, Adv. Differential Equations 10 (2005), 505–526.

[3] G. Dafni, *A div-curl decomposition for the local Hardy space*, Proceedings of the American Mathematical Society 137 (2009), 3369-3377.

[4] G. Hoepfner, J. Hounie and T. Picon, *Div-curl type estimates for elliptic systems of complex vector fields*, J. Math. Anal. Appl., Vol. 429 (2015), 774-799.

## Nicholas Braun Rodrigues - UFSCar

**Title:** Regularity for CR sections on Levi nondegenerate CR manifolds.

**Abstract:** In this work we generalize the concept of abstract CR bundles over an abstract CR manifold to $\mathcal{V}$-bundles over manifolds endowed with an involutive structure $\mathcal{V}$. We also generalize a result due to Shiferaw Berhanu and Ming Xiao regarding the microlocal regularity of CR functions on Levi nondegenerate CR manifolds to the setting of $\mathcal{V}$-bundles.

## Gabriel Mendes do Valle - UNICAMP

**Title:** Global solvability of a class of involutive systems.

**Abstract:** The present work is a review of [BP], where the authors consider a class of involutive systems of real vector fields on the $(n+1)$-torus given by

$$L_j = \partial_{t_j} + a_j(t)\partial_x, \quad j = 1, \ldots, n, \quad (t, x) \in \mathbb{T}^n \times \mathbb{T}, \quad a_j \in C^\infty(\mathbb{T}^n)\,.$$

They prove that global solvability on the $p$-th level ($0 \leqslant p \leqslant n-1$) of the induced chain complex is determined by a Liouville-type algebraic condition on the 1-form $a(t) = \sum_j a_j(t)dt_j$.

[BP] Bergamasco, A.P., Petronilho G. *Global Solvability of a Class of Involutive Systems.* Journal of Mathematical Analysis and Applications 233, no. 1 (May 1999): 314–27. https://doi.org/10.1006/jmaa.1999.6310.