10th Workshop on GEOMETRIC ANALYSIS of PDEs and SEVERAL COMPLEX VARIABLES

Program and book of abstracts

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August 5 – 9, 2019
Serra Negra, SP, Brazil

Monday, August 5

9:00 to 9:50 – Peter Ebenfelt

Title: Embeddable deformations of the standard CR structure on the 3-sphere

Abstract: By work of Kohn and Boutet de Monvel, we know that any compact strictly pseudoconvex CR manifold M of hypersurface type is embeddable in some $\mathbb{C}^N$ if $\dim M \geq 5$. For $\dim M = 3$, the situation is more subtle. It is well known that "most" (even real-analytic ones) are not embeddable. In this talk, we shall consider small deformations of the standard CR structure on S^3 and characterize the embeddable ones by an exponential map from the space of linearized embeddable deformations.

10:00 to 10:20 – Coffee break

10:20 to 11:10 – Gerardo Mendoza

Title: Nondegenerate CR structures of CR codimension > 1

Abstract: Let $\mathcal{M}$ be a smooth manifold, let $\mathcal{V} \subset \mathbb{C}T\mathcal{M}$ be an involutive subbundle defining a CR structure on $\mathcal{M}$ (as $(1,0)$ vectors), of CR codimension $r > 1$, let $\Sigma \subset T^*\mathcal{M}$ be the characteristic variety; thus Σ is a real vector bundle of rank r. Assume the CR structure is nondegenerate everywhere, which means that the Levi form of $\mathcal{V}$ at each $\theta \in \Sigma \setminus 0$ is nondegenerate. Then $\operatorname{rk} \mathcal{V}$ is even, so the rank of the horizontal bundle $\mathcal{H}$ of the CR structure is a multiple of 4 (as opposed to the case of that of a CR manifold of hypersurface type, which is just even). Furthermore, the signature of the Levi form at each $\theta \in \Sigma \setminus 0$ vanishes. In addition to proving these elementary facts I will show that the $\overline{\partial}_b$ operator is hypoelliptic on functions and provide some examples.

11:20 to 12:10 – Alex Himonas

Title: On the Cauchy problems for evolution equations with analytic initial data

Abstract: In this talk, analyticity properties in the spatial and time variables for solutions to the Cauchy problem of evolution equations with analytic initial data will be discussed. In particular, lower bound estimates for the uniform radius of spatial analyticity will be presented for several important equations including the Korteweg-de Vries and the Camassa-Holm equations. The talk is based on works with Professors S. Selberg, H. Kalisch, G. Petronilho and R. Barostichi.

12:20 to 15:00 – Lunch

15:00 to 15:20 – Tiago Picon

Title: Hardy-Littlewood-Sobolev inequalities for elliptic and canceling differential operators

Abstract: Let $A(D)$ be an elliptic homogeneous linear differential operator of order ν on $\mathbb{R}^N$, $N \geq 2$, from a complex vector space E to a complex vector space F. In this lecture we show that if $\ell \in \mathbb{R}_+$ satisfies $0 < \ell < N$ and $\ell \leq \nu$, then the estimate

$$\left(\int_{\mathbb{R}^N} |(-\Delta)^{(\nu-\ell)/2} u(x)|^q |x|^{-N+(N-\ell)q} \, dx \right)^{1/q} \leq C \|A(D)u\|_{L^1}$$

holds for every $u \in C_c^\infty(\mathbb{R}^N; E)$ and $1 \leq q < \frac{N}{N-\ell}$ if and only if $A(D)$ has the canceling property i.e., $\bigcap_{\xi \in \mathbb{R}^N \setminus \{0\}} A(\xi)[E] = \{0\}$. We also present a local version of the inequality for general linear differential operators with complex smooth variables coefficients. Variants and applications are presented, in particular (local setup) to operators associated to elliptic systems of complex vector fields.

This is joint work with Jorge Hounie (UFSCar – Brazil) and (working in progress) with Pablo De Nápoli (UBA – Argentina).

15:30 to 16:20 – Irina Mitrea

Title: Elliptic Boundary Problems in Infinitesimally Flat Uniformly Rectifiable Domains

Abstract: In this talk I will discuss recent results regarding the question of determining when singular integral operators of double layer type have small operator norms. The operator small norm estimates permit us to then establish solvability results for boundary value problems for a class of weakly elliptic second-order systems in sufficiently flat uniformly rectifiable domains, with boundary data from a library of function spaces. This includes Muckenhoupt weighted Lebesgue spaces, Lorentz spaces, Morrey spaces, and from Sobolev spaces naturally associated with these scales. This is joint work with J. J. Marin, J. M. Martell, D. Mitrea and M. Mitrea

16:30 to 16:50 – Coffee break

16:50 to 17:40 – Yunus Zeytuncu

Title: Spectral Theory of Kohn Laplacians on CR manifolds

Abstract: In this talk, we present recent results on the Kohn Laplacian on CR manifolds. We also talk about the spectrum of the Kohn Laplacian and revisit Folland's classical eigenvalue computation on the sphere. We also look at the growth rate of the eigenvalue counting function in this context. Finally, we present some computational results that lead to an understanding of spectra of more general second-order differential operators on abstract CR manifolds, including the Rossi sphere and $\mathbb{S}^7$.

17:50 to 18:40 – Andrew Raich

Minicourse: Introduction to CR Geometry and the Tangential Cauchy-Riemann Operator. Lecture 1: An Introduction to CR Geometry

Abstract: In this lecture, I will discuss boundary values of holomorphic functions and how it naturally leads to the tangential Cauchy-Riemann operator. As part of the discussion, I will review tangential vector fields and bundles, holomorphic and anti-holomorphic vector fields and bundles. For a hands-on calculation, I will give an extrinsic and concrete calculation of both $T^{1,0}(M)$ and $\bar{\partial}_b$ for (p,q)-forms where M is either the sphere S^{d-1} or the Heisenberg group.

Tuesday, August 6

9:00 to 9:50 – Shif Berhanu

Title: Unique continuation at the boundary for a class of elliptic equations

Abstract: We will present some recent results on unique continuation at the boundary for a class of second order elliptic partial differential equations and the biharmonic operator. The works are inspired by a theorem of X. Huang et al for holomorphic functions as well as generalizations by M. S. Baouendi and L. Rothschild for harmonic functions.

10:00 to 10:20 – Coffee break

10:20 to 11:10 – Andrew Raich

Title: Distributions with Decay and Restriction Problems

Abstract: In this talk, I will discuss problems in the global analysis of functions and operators and how they led us to define distributions with decay and the Fourier microglobal wavefront set. As an application, I will show how these new ideas lead to a new twist on the classical restriction problem in harmonic analysis. This work is joint with Professor Gustavo Hoepfner from the Universidade Federal de São Carlos.

11:20 to 12:10 – Makhlouf Derridj

Title: Gevrey regularity of Gevrey vectors of second order partial differential operators with non negative characteristic form

Abstract: We study the s'-Gevrey regularity of s-Gevrey vectors of a second order partial differential operator P with non negative characteristic form on an open set U (studied by O. Oleinik and E. V. Radkevic concerning hypoellipticity), giving a close relation between s and s'; more precisely we associate to any open subset V of U what we call its type with respect to P (integer, finite or not) denoted $t(V;P)$. Then we prove the following:

Theorem. *If the coefficients of terms of order* 1 *and* 2 *of* P *are real, all coefficients are in* s*-Gevrey class in* V *and* $t(V;P)$ *finite then any* s*-Gevrey vector of* P *in* V *is in* s'*-Gevrey class in* V *, with* $s' = t(V;P)\cdot s$*. Moreover,* s' *is optimal.*

12:20 to 15:00 – Lunch

15:00 to 15:20 – Renato A. Laguna

Title: Global hypoellipticity of planar vector fields

Abstract: In [1] we examined the global hypoellipticity (GH) of complex, nonsingular, $\mathcal{C}^\infty$ vector fields L on $\mathbb{R}^2$.

In the cases where L has only one Sussmann orbit, we give a geometric condition which, together with $(\mathcal{P})$ of Nirenberg and Treves, characterizes GH. This generalizes Theorem 3.1 from Hounie's [2], which dealt with the cases where

$$L = g(x,t)\big[\partial_t + ib(x,t)\partial_x\big], \qquad |g| > 0, \quad \Im b \equiv 0.$$

We also show, still assuming that L has only one orbit, that the existence of a global first integral is necessary for GH.

When L has more than one orbit, we can rule out GH if L is real, essentially real, or real-analytic, as well as in many other cases that are described in terms of the Sussmann orbits.

Joint work with Sérgio Luís Zani and Adalberto P. Bergamasco.

Bibliography

[1] Adalberto P. Bergamasco, Renato A. Laguna, and Sérgio L. Zani. Global hypoellipticity of planar complex vector fields. *Journal of Differential Equations*, 2019. ISSN: 0022-0396. DOI: https://doi.org/10.1016/j.jde.2019.05.027. URL: http://www.sciencedirect.com/science/article/pii/S002203961930244X.

[2] Jorge Hounie. Global Cauchy problems modulo flat functions. *Advances in Mathematics*, 51(3):240–252, Mar. 1984. ISSN: 0001-8708. DOI: https://doi.org/10.1016/0001-8708(84)90009-4. URL: http://www.sciencedirect.com/science/article/pii/0001870884900094.

15:30 to 16:20 – Giuseppe Della Sala

Title: Deformations of CR maps

Abstract: In joint work with Bernhard Lamel and Michael Reiter, we study the deformation theory of maps of a CR manifold into a family of CR manifolds of higher dimension. As an offshoot, we obtain structural results on the locus of points admitting a CR map from a given manifold.

16:30 to 16:50 – Coffee break

16:50 to 17:40 – Alberto Parmeggiani

Title: On the solvability of a class of second order degenerate operators

Abstract: In this talk I will be concerned with the problem of solvability of second order degenerate operators that are not of principal type. I will describe some recent results, obtained in joint work with S. Federico, about local solvability in the Sobolev spaces of a class of degenerate operators which is an elaboration of the class considered by Colombini-Cordaro-Pernazza (in turn, an elaboration of the adjoint of the Kannai operator).

17:50 to 18:40 – Andrew Raich

Minicourse: Introduction to CR Geometry and the Tangential Cauchy-Riemann Operator. Lecture 2: The tangential Cauchy-Riemann operator and the Levi form

Abstract: In this lecture, I have several goals. First, I will show what type of structure is needed on a manifold in order for $\bar{\partial}_b$ to be defined and give many examples. I will also present an abstract (that is, intrinsic) way to formulate $\bar{\partial}_b$ on a general CR manifold. Second, I will contrast the behavior of $\bar{\partial}_b$ with the more familiar operator $\bar{\partial}$. For example, $\bar{\partial}$ commutes with the pullbacks of holomorphic mappings while $\bar{\partial}_b$ does not. I will also remind people of the definitions and properties of pullbacks and pushforwards. Finally, I will show how the CR geometric structure forms the prototype for Involutive Structures to connect the geometry discussed in these lectures with a structure with which many of the Brazilian graduate students attending the lectures will be familiar.

Wednesday, August 7

9:00 to 9:50 – François Treves

Title: Trépreau's Theorem about Condition (Ψ)

Abstract: In his thesis (1984) J. M. Trépreau proved the equivalence of the **microlocal** solvability of an analytic pseudodifferential operator of principal type, P, and the validity for P of the so-called Condition (Ψ). The proof of the equivalence of (Ψ) with the **local** solvability of a C^∞ pseudodifferential operator of principal type was completed by N. Dencker in 2006. The proof of Trépreau's Theorem is based on the theory of SVC (Several Complex Variables) and the method of quantized transformations introduced by M. Sato in the late 1950s: P is (microlocally!) transformed into $\partial/\partial z_1$ acting on the ring of holomorphic functions in the neighborhood of a boundary point, in a strictly pseudoconvex domain Ω with analytic boundary – with no restriction on the growth of the functions at the boundary. The transformation specifies the choice of the domain Ω and reflects, in the defining equation $\rho(z) < 0$ of Ω at the boundary point, the intrinsic properties of the principal symbol of P embodied in (Ψ). A natural question from the SVC viewpoint is how this translates in terms of the the Chern-Moser-Tanaka invariants. The lecture will explain as simply as possible the main points of Trépreau's proof and, in parallel, show how some of the steps can be naturally extended to an involutive system of analytic pseudodifferential operators of principal type, perhaps opening the way to the study of the microlocal exactness, at a given degree, of the associated differential complex; and, most tantalizingly, to finding out what is the analogue of (Ψ) for exactness at that degree.

10:00 to 10:20 – Coffee break

10:20 to 11:10 – Laurent Stolovitch

Title: TBA

Abstract: TBA.

11:20 to 12:10 – Nordine Mir

Title: Regularity of CR maps of abstract CR structures

Abstract: In this joint work with B. Lamel, we study the $\mathcal{C}^\infty$ regularity problem for CR maps from an abstract CR manifold M into some complex euclidean space $\mathbb{C}^{N'}$. We show that if M satisfies a certain condition called the microlocal extension property, then any $\mathcal{C}^k$-smooth CR map $h\colon M \to \mathbb{C}^{N'}$, for some integer k, which is nowhere $\mathcal{C}^\infty$-smooth on some open subset Ω of M, has the following property: for a generic point q of Ω, there must exist a formal complex subvariety through $h(q)$, tangent to $h(M)$ to infinite order, and depending in a $\mathcal{C}^1$ and CR manner on q. As a consequence, we obtain several $\mathcal{C}^\infty$ regularity results generalizing earlier ones by Berhanu-Xiao and the authors (in the embedded case).

12:20 to 14:30 – Lunch

14:30 to 15:20 – Paulo D. Cordaro

Title: TBA

Abstract: TBA.

15:30 to 17:00 – Poster session

We invite everyone to come and prestige the works presented in our poster session!

Thursday, August 8

9:00 to 9:50 – Xiaojun Huang

Title: Hyperbolic metrics on Stein spaces with compact spherical boundaries

Abstract: This is a joint work with Ming Xiao. We discuss how to construct a hyperbolic metric over a Stein space with spherical boundary. The technique we use is to employ holomorphic continuation along curves.

10:00 to 10:20 – Coffee break

10:20 to 11:10 – Abdelhamid Meziani

Talk: On Isometric Deformations of Two Dimensional Surfaces

Abstract: We will discuss isometric deformations of two dimensional surfaces in $\mathbb{R}^3$. The surface is assumed to have boundary, nonnegative curvature and with genus zero. We will also highlight the role played by asymptotic curves to rigidify the surface.

11:20 to 12:10 – Florian Bertrand

Title: On the defect of stationary discs and jet determination problems

Abstract: I will discuss some recent results on the links between stationary discs introduced by Lempert, the defect of analytic discs, and jet determination of CR automorphisms of nondegenerate – in a sense to be defined – real submanifolds of $\mathbb{C}^N$. This is joint work with Francine Meylan.

12:20 to 15:00 – Lunch

15:00 to 15:20 – Luis F. Ragognette

Title: On microlocal analysis of hyperfunctions

Abstract: In 2012, Shiferaw Berhanu and Jorge Hounie introduced a class of FBI transforms and proved that it can be used to characterize the microlocal analyticity and microlocal smoothness of distributions. This inspired other works where different types of microlocal regularities were consider.

In this joint work with Gustavo Hoepfner, we obtain a subclass of these FBI transforms that can be applied on hyperfunctions and introduced some techniques to study the wavefront set of a hyperfunction when we consider several functional analytic spaces, among them the space of smooth, real-analytic, Gevrey functions.

15:30 to 16:20 – Howard Jacobowitz

Title: A Conjecture of Trautman

Abstract: In 1998 the physicist Andrzej Trautman conjectured:

> *A three-dimensional CR manifold is locally realizable if and only if its canonical bundle admits a closed nowhere zero section.*

A real three-dimensional hypersurface in the two-dimensional complex space C^2 is said to be a realizable CR manifold. The definition of a CR structure on an abstract three-dimensional manifold is essentially due to E. Cartan. A natural question is then: When can an abstract CR manifold be realized by a real hypersurface in C^2.

In this talk we review the relevant mathematical definitions. Then we outline the construction due to Robinson and Trautman of the abstract CR manifold associated to an optical geometry, that is, to a shear-free congruence of null geodesics in a Lorentzian 4-space. A special case of the conjecture, already known in a different context, will be proved.

16:30 to 16:50 – Coffee break

16:50 to 17:40 – Jiří Lebl

Title: Severi's theorem for codimension 2 CR singular manifolds $\mathbb{C}^3$

Abstract: Locally, every real-analytic CR function on a real-analytic CR submanifold is a restriction of a holomorphic function. Such a theorem does not hold for all CR singular manifolds. For codimension 2 CR singular submanifolds in 3 complex dimensions, we find the exact condition on second order data giving the extension result (so a complete characterization for quadrics). In particular, generically Severi's theorem holds in this setting. This is a joint work with Alan Noell and Sivaguru Ravisankar.

17:50 to 18:40 – Andrew Raich

Minicourse: Introduction to CR Geometry and the Tangential Cauchy-Riemann Operator. Lecture 3: The $\bar{\partial}_b$-problem and the Heisenberg group

Abstract: In this lecture, I will introduce the $\bar{\partial}_b$-problem and review the beautiful calculations of Folland and Stein ('74) on the Heisenberg group in which they completely solve the $\bar{\partial}_b$-problem.

Friday, August 9

9:00 to 9:50 – John D'Angelo

Title: Rational CR maps between spheres: A link to number theory

Abstract: I will give my N plus second talk on rational sphere maps. We begin by showing why the study of rational sphere maps should regard the target dimension as a variable. Then we will consider certain combinatorial number-theoretic aspects that arise when one seeks mappings with specific target dimension. Some of this work was done jointly with Lebl a few years ago and the talk will also mention computations done by Lebl-Lichtblau. For example, I will try to explain how one proves that uniqueness of sharp polynomials fails when the degree is congruent to 1, 3, 7, or 11 mod 12. There will be several new observations and questions.

10:00 to 10:20 – Coffee break

10:20 to 11:10 – Ilya Kossovskiy

Title: Real-analytic coordinates for smooth strictly pseudoconvex CR-structures

Abstract: For a smooth strictly pseudoconvex hypersurface in a complex manifold, we give a necessary and sufficient condition for being CR-diffeomorphic to a real-analytic CR manifold. Our condition amounts to a holomorphic extension property for the canonically associated function expressing 2-jets of the formal Segre varieties in terms of their 1-jets. We also express this condition in equivalent terms for a Fefferman type determinant. This work is joint with Dmitri Zaitsev.

11:20 to 12:10 – Francine Meylan

Title: Polynomial models in CR geometry

Abstract: Let $M \subset \mathbb{C}^{n+1}$ be a real smooth hypersurface given near p by

$$\mathsf{Im}w = P(z,\bar{z}) + o(|z|^d, \mathsf{Re}w), \quad (z,w) \in \mathbb{C}^n \times \mathbb{C},$$

where P is a real homogeneous polynomial without pluriharmonic terms of degree $d \geq 2$.

Let M_P be its associated polynomial model real hypersurface given by

$$\mathsf{Im}w = P(z,\bar{z}).$$

In this talk we discuss (and motivate) the classification of polynomial models M_P associated to real hypersurfaces $M \subset \mathbb{C}^{n+1}$ according to $hol(M_P, p)$, the set of holomorphic vector fields in $\mathbb{C}^{n+1}$ whose real parts are tangent to M_P.

We illustrate it on a class of concrete examples, polynomial models in $\mathbb{C}^3$, of the form

$$\mathsf{Im}w = P(z)\overline{Q(z)} + Q(z)\overline{P(z)},$$

where P and Q are holomorphic polynomials in $z \in \mathbb{C}^2$.

12:20 to 15:00 – Lunch

15:00 to 15:20 – Giuliano Zugliani

Title: Tube structures of corank 1 with forms defined on compact surfaces

Abstract: We study a locally integrable structure of tube type and corank 1 by considering a linear partial differential operator $\mathbb{L}$ associated to a smooth closed 1-form c defined on a smooth closed n-manifold. The main result characterizes the global solvability and the global hypoellipticity of $\mathbb{L}$ when $n = 2$ in terms of geometric properties of a primitive of a convenient exact pullback of the form $\mathfrak{Im}(c)$ as well as in terms of homological properties of $\mathfrak{Re}(c)$ related to small divisors phenomena. Although the full characterization is restricted to surfaces, some particular results will hold true for any dimension. We also obtain informations on the global solvability of $\mathbb{L}^{n-1}$ – the last nontrivial operator of the complex when M is orientable.

Joint work with Jorge Hounie from UFSCar. We thank FAPESP for the financial support.

15:30 to 16:20 – Antonio Bove

Title: Analytic Regularity for Sums of Squares with a Low Dimensional Characteristic Variety

Abstract: We study the real analytic or Gevrey regularity for solutions to sums of squares of vector fields with real analytic coefficients. We give the Gevrey regularity for a class of monomial coefficients vector fields when the characteristic variety has dimension 2. We prove an optimality result for the generalized Métivier operator and for an operator in the class considered above. In both cases the characteristic manifold has dimension 2.

16:30 to 16:50 – Coffee break

16:50 to 17:40 – Bernhard Lamel

Title: TBA

Abstract: TBA.

17:50 to 18:40 – Andrew Raich

Minicourse: Introduction to CR Geometry and the Tangential Cauchy-Riemann Operator. Lecture 4: CR Geometry and $\bar{\partial}_b$ today

Abstract: In the final lecture, I will outline several open problems in CR Geometry and the study of $\bar{\partial}_b$. Problems included in this list may include:

1. The L^2 and L^2-Sobolev theory of the $\bar{\partial}_b$-problem, including pseudoconvexity, compactness, finite type, and condition $Y(q)$.
2. Techniques to solve the $\bar{\partial}_b$-problem on spaces other than L^2.
3. Problems in the existence (or non-existence) of CR mappings between CR manifolds.