Workshop on Submanifold Theory and Geometric Analysis

UFSCar, São Carlos, Brazil, August 05 – 09, 2019

Tuesday- 14h - 14:50h -Auditório do DM

Luis J. Alías

(Universidad de Murcia - Spain)

The principal curvature theorem and its applications to constant mean curvature hypersurfaces in euclidean space

Abstract. The so called Principal Curvature Theorem (PCT) is a purely geometric result on the principal curvatures of complete hypersurfaces in Euclidean space given by Smyth and Xavier (*Invent. Math.* 90:443–450, 1987) in their proof of Efimov's theorem in dimension greater than two. As another application of the PCT, they also proved that the only complete hypersurfaces immersed in $\mathbb{R}^{n+1}$ with constant mean curvature $H \neq 0$ and having non-positive Ricci curvature are the right circular cylinders of the form $\mathbb{R}^{n-1} \times \mathbb{S}^1(r)$, with $r > 0$, extending to the n-dimensional case a previous result for $n = 2$ due to Klotz and Osserman. In this lecture we will introduce new applications of the PCT to the study of complete hypersurfaces with constant mean curvature immersed into the Euclidean space $\mathbb{R}^{n+1}$, and, more generally, with constant higher order mean curvature. For instance, among other results, we will prove that if Mn is a complete hypersurface in $\mathbb{R}^{n+1}$ $(n \geq 3)$ with constant mean curvature $H \neq 0$ and having two distinct principal curvatures, one of them being simple, then $\sup_M Scal \geq 0$ and equality holds if and only if M is a right circular cylinder $\mathbb{R}^{n-1} \times \mathbb{S}^1(r)$, with $r > 0$. Similarly, we will also prove that if M^n is a complete hypersurface in $\mathbb{R}^{n+1}$ with constant k-th mean curvature $H_k \neq 0$ $(2 \leq k < n)$ and two distinct principal curvatures, one of them being simple, and its Gauss-Kronecker curvature K does no change sign, then $K = 0$ and M is a cylinder $\mathbb{R} \times \mathbb{S}^{n-1}(r)$, with $r > 0$. Our results in this talk are part of our joint work with S. Carolina García Martínez (*Geom. Dedicata* 156:31–47, 2012) and with Josué Meléndez (*Geom. Dedicata* 182:117–131, 2016; *Geom. Dedicata* 199:273–280, 2019).

Support: FAPESP CAPES CNPq SBM

Organizers: UFSCar DM ufscar INCTMat ICMC USP São Carlos