Workshop on Submanifold Theory and Geometric Analysis

UFSCar, São Carlos, Brazil, August 05 – 09, 2019

Monday- 9:30h - 10:20h -Auditório do DM

Carlos Olmos

(Córdoba - Argentina)

Homogeneous Riemannian manifolds with non-trivial nullity

Abstract. The main purpose of this lecture is to introduce to techniques in homogeneous geometry based in geometric constructions and Killing fields rather than the usual Lie algebra point of view. For this purpose we will speak about a classical problem in Riemannian geometry, which goes back to Chern-Kuiper, which is the study of the nullity distribution of the curvature tensor. For homogeneous Riemannian manifolds nothing was known about this problem, not even a non-trivial example. In a recent work with A.J. Di Scala and F. Vittone we developed a general theory for homogeneous spaces $M = G/H$ in relation with the nullity. We construct natural invariant (different and increasing) distributions associated with the nullity, that give a deep insight of such spaces. In particular, there must exist an order-two transvection, not in the nullity, with null Jacobi operator. This fact was very important for finding out the first homogeneous examples with non-trivial nullity, i.e. where the nullity distribution is not parallel. Moreover, there are irreducible examples of conullity $k = 3$, the smallest possible, in any dimension. We also proved that H is trivial and G is solvable if $k = 3$ (and it is an open problem whether G must be always solvable) The leaves of the nullity of M are closed (this involves a delicate argument). This implies that M is a Euclidean affine bundle over the quotient by the leaves of the nullity. Moreover, the perpendicular distribution of the nullity defines a metric connection on this bundle with transitive holonomy or, equivalently, a completely non-integrable distribution (this is not in general true for an arbitrary autoparallel and flat invariant distribution). There are some general obstruction for the existence of non-trivial nullity: e.g., if G is reductive (in particular, if M is compact), or if G is two-step nilpotent.

Support: FAPESP CAPES CNPq SBM

Organizers: ufscar DM ufscar INCTMat ICMC USP São Carlos