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DEPARTAMENTO DE MATEMÁTICA

COLÓQUIO

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Falará sobre:

The stochastic Weiss conjecture

Resumo. The semigroup approach to stochastic partial differential equations (SPDEs) consists in to formulate such equations as abstract evolution equations of the form

$$\begin{cases} dU(t) = AU(t)\,dt + B\,dW_H(t), & t \geq 0, \\ U(0) = x, \end{cases} \tag{1}$$

where A is the generator of a strongly continuous semigroup $(S(t))_{t\geq 0}$ of linear operators on a Banach space E, W_H is a cylindrical Brownian motion over a Hilbert space H and B is a bounded linear operator from H to E. Here E is the state space of the equation, say $L^p(\mathcal{O})$ with $\mathcal{O} \subset \mathbf{R}^d$ open and bounded, and B 'injects' noise into E.
As it is well known, the transition semigroup $(P(t))_{t\geq 0}$ defined by

$$P(t)f(x) = \mathbf{E}f\big(U(t,x)\big), \quad f \in C_b(E),$$

is an important tool in the study of the long time behaviour of the system, and a fundamental question in this context is that of the existence of an invariant measure μ for $(P(t))_{t\geq 0}$, since this allows its extension to $L^p(E,\mu)$ through a standard functional-analytic argument. For stochastic linear evolution equations, the *stochastic Weiss conjecture* is the assertion that, under suitable assumptions on A, B and E, the existence of an invariant measure for (1) is equivalent to a condition on the operator-valued function $\lambda \mapsto \lambda^{1/2}R(\lambda, A)B$. Here $R(\lambda, A) = (\lambda - A)^{-1}$ is the resolvent of A at λ.
In this talk we sketch the solution [1] of this problem, where it is proved that if $-A$ has a bounded H^∞-calculus of angle less than $\pi/2$ on a Banach space E having the so called Pisier property (α), then the existence of an invariant measure for (1) is equivalent to the convergence in probability of the Gaussian series

$$\sum_{n\in\mathbf{Z}} \gamma_n 2^{n/2} R(2^n, A)B$$

in the space $\gamma(H, X)$ of γ-radonifying operators from H to E.
In the end, we will discuss some open problems related to this topic which, from our present point of view, may lead to further interesting developments. This is a joint work with Bernhard Haak and Jan van Neerven.

[1] J. G. de Abreu Jr., B. H. Haak and J. M. A. M. van Neerven, The stochastic Weiss conjecture for bounded analytic semigroups, *J. London Math. Soc.* (2) **88** (2013) 181-201 .

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