A CAUCHY-KOVALEVSKY THEOREM FOR NONLINEAR AND NONLOCAL EQUATIONS

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Abstract. For a generalized Camassa-Holm equation it is shown that the solution to the Cauchy problem with analytic initial data is analytic in both variables, locally in time and globally in space. Furthemore, an estimate for the analytic lifespan is provided. To prove these results, the equation is written as a nonlocal autonomous differential equation on a scale of Banach spaces and then a version of the abstract Cauchy-Kovalevsky theorem is applied, which is derived by the power series method in these spaces. Similar abstract versions of the nonlinear Cauchy-Kovalevsky theorem have been proved by Ovsyannikov, Treves, Baouendi and Goulaouic, Nirenberg, and Nishida.

In memory of M. Salah Baouendi

1. Introduction and Results

For k any positive integer and b any real number, we consider the Cauchy problem for the following generalized Camassa-Holm equation (g-kbCH)

$$u_t = (1-\partial_x^2)^{-1}\big[u^k u_{xxx} + bu^{k-1}u_x u_{xx} - (b+1)u^k u_x\big], \qquad u(0) = u_0, \tag{1.1}$$

and prove that if the initial datum u_0 is analytic on the line or the torus, then the solution is analytic in both variables, globally in x and locally in t. This should be contrasted with the KdV equation, whose solution is analytic in x but not in t when the initial data are analytic (see [Tru], [GH]). Well-posedness in the sense of Hadamard of the initial value problem for this equation in Sobolev spaces has been proved in [HH3]. More precisely, there it was proved that if $s > 3/2$ and $u_0 \in H^s$ then there exists $T > 0$ and a unique solution $u \in C([0,T]; H^s)$ of the initial value problem for g-kbCH which depends continuously on the initial data u_0. Furthermore, we have the estimate

$$\|u(t)\|_{H^s} \le 2\|u_0\|_{H^s}, \quad \text{for} \quad 0 \le t \le T \le \frac{1}{2kc_s\|u_0\|_{H^s}^k}, \tag{1.2}$$

where $c_s > 0$ is a constant depending on s. Also, the data-to-solution map is not uniformly continuous from any bounded subset in H^s into $C([0,T]; H^s)$. Concerning global solutions, it was shown in [HT] that if $u_0 \in H^s$, $s > 3/2$, and $m_0 = (1-\partial_x^2)u_0$ does not change sign on $\mathbb{R}$, then the solution to the Cauchy problem for g-kbCH persists for all time in the case $b = k+1$.

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Furthermore, in the cases that $b = k$ with k a positive odd number or $k = 1$ and $b \in [0, 3]$ this equation exhibits unique continuation properties.

The g-kbCH equation, besides having interesting analytic properties, it also contains two integrable equations with quadratic nonlinearities. The first is the well known Camassa-Holm equation (see [CH], [FF], [F])

$$u_t = (1 - \partial_x^2)^{-1}\big[uu_{xxx} + 2u_xu_{xx} - 3uu_x\big], \tag{1.3}$$

which is obtained from (1.1) by letting $k = 1$ and $b = 2$, and the second is the Degasperis-Procesi equation [DP]

$$u_t = (1 - \partial_x^2)^{-1}\big[uu_{xxx} + 3u_xu_{xx} - 4uu_x)\big], \tag{1.4}$$

which is obtained from (1.1) by letting $k = 1$ and $b = 3$. Also, for $k = 2$ and $b = 3$ it gives the Novikov equation [N]

$$u_t = (1 - \partial_x^2)^{-1}\big[u^2u_{xxx} + 3uu_xu_{xx} - 4u^2u_x\big], \tag{1.5}$$

which is an integrable equation with cubic nonlinearities.

Integrable equations possess many special properties including a Lax pair, a bi-Hamiltonian formulation, and they can be solved by the Inverse Scattering Method. Also, they possess infinitely many conserved quantities. The H^1-norm of a solution u is such a quantity for the Camassa-Holm and the Novikov equations, since it can be shown that

$$\frac{d}{dt}\|u(t)\|_{H^1}^2 = \frac{d}{dt}\int_{\mathbb{R} \text{ or } \mathbb{T}} \Big[u^2(t) + u_x^2(t)\Big]\, dx = 0. \tag{1.6}$$

In fact, this quantity is conserved for all members of g-kbCH with $b = k + 1$.

Another interesting property of the g-kbCH equation is that it possesses peakon-type solitary wave solutions [GH]. On the line, these solutions are of the form

$$u(x,t) = c^{1/k}e^{-|x-ct|},$$

where $c > 0$ is the wave speed. On the circle, these solutions take the form

$$u(x,t) = \frac{c^{1/k}}{\cosh(\pi)}\cosh([x - ct]_p - \pi),$$

where

$$[x - ct]_p \doteq x - ct - 2\pi\left[\frac{x - ct}{2\pi}\right].$$

In this work we study the Cauchy problem for the g-kbCH equation for initial data in spaces of analytic functions. More precisely, the initial data belong in the following scale of decreasing Banach spaces. For $\delta > 0$ and $s \geq 0$, in the periodic case they are defined by

$$G^{\delta,s}(\mathbb{T}) = \{\varphi \in L^2(\mathbb{T}) : ||\varphi||^2_{G^{\delta,s}(\mathbb{T})} \doteq ||\varphi||^2_{\delta,s} = \sum_{k\in\mathbb{Z}}(1 + |k|)^{2s}e^{2\delta|k|}|\widehat{\varphi}(k)|^2 < \infty\}, \tag{1.7}$$

while in the nonperiodic case they are defined by

$$G^{\delta,s}(\mathbb{R}) = \{\varphi \in L^2(\mathbb{R}) : ||\varphi||^2_{G^{\delta,s}(\mathbb{R})} \doteq ||\varphi||^2_{\delta,s} = \int_{\mathbb{R}}(1 + |\xi|)^{2s}e^{2\delta|\xi|}|\widehat{\varphi}(\xi)|^2 d\xi < \infty\}. \tag{1.8}$$

Here, when a result holds for both the periodic and non-periodic case then we use the notation $||\cdot||_{\delta,s}$ for the norm and $G^{\delta,s}$ for the space in both cases. We observe that a function φ in $G^{\delta,s}(\mathbb{T})$ has an analytic extension to a symmetric strip around the real axis with width δ (see Lemma 1). This δ is called the *radius of analyticity* of φ.

Next, we state the main result of this work. For the sake of simplicity we shall assume that our initial data u_0 belong in $G^{1,s+2}$.

Theorem 1. *Let $s > \frac{1}{2}$. If $u_0 \in G^{1,s+2}$ on the circle or the line, then there exists a positive time T, which depends on the initial data u_0 and s, such that for every $\delta \in (0,1)$, the Cauchy problem* (1.1) *has a unique solution u which is a holomorphic function in $D(0, T(1-\delta))$ valued in $G^{\delta,s+2}$. Furthermore, the analytic lifespan T satisfies the estimate*

$$T \approx \frac{1}{||u_0||^k_{1,s+2}}. \tag{1.9}$$

A more precise statement of estimate (1.9) is provided in Section 4 (see (4.6)). For the Camassa-Holm equation on the circle, a result similar to Theorem 1 but without an analytic lifespan estimate like (1.9) was proved in [HM1]. Furthermore, for the Camassa-Holm, the Degasperis-Procesi and the Novikov equations Theorem 1 was proved in [BHP]. The present research note generalizes this result to the g-kbCH equation using very similar techniques. We mention here that all this work was motivated by the Cauchy-Kovalevsky type result for the Euler equations that was proved by Baouendi and Goulaouic in [BG2] as an application of a more general theory about analytic pseudo-differential operators. For more information about nonlinear versions of the Cauchy-Kovalevsky theorem, we refer the reader to Ovsyannikov [O1], [O2], [O3], Treves [Tre1], [Tre2], Baouendi and Goulaouic [BG1], Nirenberg [Nr], and Nishida [Ns].

Finally, we mention that there is an extensive literature about Camassa-Holm type equations. For results about well-posedness, continuity properties and traveling wave solutions for these and related evolution equations, we refer the reader to [CHT], [CL], [CM], [CS], [DHH], [D], [DHH], [HH1] [HH2], [HH3], [HHG] [HK], [HKM], [HM1], [HM2], [HM3], [HMP], [HoH], [HLS], [M], [MN], [L], [LO], [RB], [Ti], [Y], and the references therein.

The paper is organized as follows. In Section 2, we state the basic properties of the $G^{\delta,s}$ spaces and their norms. Then, in Section 3 we use the power series method to provide a version of an autonomous Ovsyannikov theorem. Finally, in Section 4 we prove Theorem 1 by using the Ovsyannikov theorem.

2. Properties of $G^{\delta,s}$ spaces

Recall that a family of Banach spaces $\{X_\delta\}_{0<\delta\le 1}$ is said to be a scale of decreasing Banach spaces if for any $0 < \delta' < \delta \le 1$ we have

$$X_\delta \subset X_{\delta'}, \; ||.||_{\delta'} \le ||.||_\delta. \tag{2.1}$$

In the following lemmas, whose proof can be found in [BHP], we summarize the basic properties of the $G^{\delta,s}$ spaces and their norms. Lemma 1 provides an alternative description of the $G^{\delta,s}$

spaces, while Lemmas 2 and 3 show that the $G^{\delta,s}$ spaces form a scale of decreasing Banach spaces and provide the tools for estimating the right hand-side of the g-kbCH equation (1.1).

Lemma 1. *Let $\varphi \in G^{\delta,s}$. Then, φ has an analytic extension to a symmetric strip around the real axis of width δ, for $s \geq 0$ in the periodic case and $s > \frac{1}{2}$ in the non-periodic case.*

Lemma 2. *If $0 < \delta' < \delta \leq 1$, $s \geq 0$ and $\varphi \in G^{\delta,s}$ on the circle or the line, then*

$$||\partial_x \varphi||_{\delta',s} \leq \frac{e^{-1}}{\delta - \delta'}||\varphi||_{\delta,s} \tag{2.2}$$

$$||\partial_x \varphi||_{\delta,s} \leq ||\varphi||_{\delta,s+1} \tag{2.3}$$

$$||(1-\partial_x^2)^{-1}\varphi||_{\delta,s+2} \leq 2||\varphi||_{\delta,s} \tag{2.4}$$

$$||(1-\partial_x^2)^{-1}\varphi||_{\delta,s} \leq ||\varphi||_{\delta,s} \tag{2.5}$$

$$||\partial_x(1-\partial_x^2)^{-1}\varphi||_{\delta,s} \leq ||\varphi||_{\delta,s}. \tag{2.6}$$

Lemma 3. *For $\varphi \in G^{\delta,s}$ on the circle or the line the following properties hold true:*

1) If $0 < \delta' < \delta$ and $s \geq 0$, then $||\cdot||^2_{\delta',s} \leq ||\cdot||^2_{\delta,s}$; i.e. $G^{\delta,s} \hookrightarrow G^{\delta',s}$.

2) If $0 < s' < s$ and $\delta > 0$, then $||\cdot||^2_{\delta,s'} \leq ||\cdot||^2_{\delta,s}$; i.e. $G^{\delta,s} \hookrightarrow G^{\delta,s'}$.

3) For $s > 1/2$ and $\varphi, \psi \in G^{\delta,s}$ we have

$$||\varphi\psi||_{\delta,s} \leq c_s||\varphi||_{\delta,s}||\psi||_{\delta,s}, \tag{2.7}$$

where $c_s = \sqrt{2(1+2^{2s})\sum_{k=0}^{\infty}\frac{1}{(1+|k|)^{2s}}}$ in the periodic case and $c_s = \sqrt{\frac{2(1+2^{2s})}{2s-1}}$ in the non-periodic case.

Remark. For $s = 1$ we obtain, in the periodic case, $c_1 = \sqrt{2(1+4)\sum_{\ell=1}^{\infty}\frac{1}{\ell^2}} = \sqrt{\frac{5\pi^2}{3}}$, and, in the non-periodic case, we have $c_1 = \sqrt{10}$.

Lemma 4. *If $u_0 \in C^{\omega}(\mathbb{T})$, there exists $\delta_0 > 0$ such that $u_0 \in G^{\delta_0,s}(\mathbb{T})$ for any $s \geq 0$.*

From now on we fix $s > 1/2$, and without loss of generality we assume that $\delta_0 = 1$.

3. The Power Series Method for the autonomous Ovsyannikov theorem

Next, following Treves [Tre1], [Tre2] and [Tre3] we provide a brief description of an autonomous Ovsyannikov theorem that we will use for the proof of Theorem 1. A more detailed exposition is contained in [BHP].

Given a decreasing scale of Banach spaces $\{X_\delta\}_{0<\delta\leq 1}$ and initial data $u_0 \in X_1$ we consider the Cauchy problem

$$\frac{du}{dt} = F(u), \quad u(0) = u_0, \tag{3.1}$$

where $F : X_0 \to X_0$ is Ovsyannikov analytic at u_0 and $X_0 = \bigcup_{0<\delta<1} X_\delta$. We recall that $F(u)$ is Ovsyannikov analytic at u_0 if there exist positive constants R, A and C_0 such that for all $k \in \mathbb{Z}_+$ and $0 < \delta' < \delta < 1$ we have

$$||D^k F(u)(v_1, \ldots, v_k)||_{\delta'} \leq \frac{AC_0^k k!}{\delta - \delta'} ||v_1||_\delta \ldots ||v_k||_\delta, \tag{3.2}$$

for all $u \in \{u \in X_\delta : ||u - u_0||_\delta < R\}$ and $(v_1, \ldots, v_k) \in X_\delta^k$, where $D^k F$ is the Frechet derivative of F of order k. Such a function can be represented by its Taylor series near u_0. More precisely, given any pair (δ, δ'), $0 < \delta' < \delta < 1$ and any $u \in B_\delta(u_0; R)$ the Taylor series

$$\sum_{k=0}^{\infty} \frac{1}{k!} D^k F(u_0) \underbrace{(u - u_0, \ldots, u - u_0)}_{k}$$

converges absolutely to $F(u)$ in $X_{\delta'}$.

The fundamental result, which we shall need for the proof of Theorem 1, reads as follows.

Theorem 2. *If $u_0 \in X_1$ and F is Ovsyannikov analytic, then there exists $T > 0$ such that the Cauchy problem (3.1) has a unique solution which, for every $\delta \in (0, 1)$ is a holomorphic function in $D(0, T(1 - \delta))$ valued in X_δ satisfying*

$$\sup_{|t|<T(1-\delta)} \|u(t) - u_0\|_\delta < R, \quad 0 < \delta < 1. \tag{3.3}$$

Moreover, the lifespan T is given by

$$T = \frac{1}{2e^2 AC_0}, \tag{3.4}$$

where the constants R, A and C_0 come from the definition of Ovsyannikov analytic function.

The proof of this result uses the power series method and it can be found in [BHP].

4. Proof of theorem 1

Next, we shall use Theorem 2 in order to prove Theorem 1 for the Cauchy problem of the g-kbCH equation (1.1). In this situation the function $F(u)$ has the following nonlocal form

$$F(u) = (1 - \partial_x^2)^{-1} \big[u^k u_{xxx} + bu^{k-1} u_x u_{xx} - (b + 1)u^k u_x\big]. \tag{4.1}$$

Also, the scale of decreasing Banach spaces is given by

$$\{G^{\delta,s+2}\}_{0<\delta\leq 1}, \quad \text{with norm} \quad ||\cdot||_{\delta,s}. \tag{4.2}$$

In order to prove the existence and uniqueness of a holomorphic solution to our Cauchy problem (1.1), by using Theorem 2, it suffices to estimate $||D^k F(u_0)(v_1, \ldots, v_k)||_{\delta'}$ for all $(v_1, \ldots, v_k) \in X_\delta^k$. This, in combination with formula (3.4), will also provide the desired estimate (1.9) for the analytic lifespan of the solution in terms of the norm of the initial data.

Next, we shall provide an estimate for $||D^k F(u_0)(v_1, \ldots, v_k)||_{\delta'}$, only for the first term of the right-hand side of F equation (4.1), that is

$$F_1(u) \doteq (1 - \partial_x^2)^{-1} [u^k \partial_x^3 u]. \tag{4.3}$$

The estimate for the other two terms is analogous. By using the following formula for the Frechet derivative of F_1 of order j, $1 \leq j \leq k$, at the point u_0,

$$D^j F_1(u_0)(v_1, \ldots, v_j) = \frac{d}{d\tau_j} \cdots \frac{d}{d\tau_1} \Big\{ F_1(u_0 + \sum_{i=1}^{j} \tau_i v_i) \Big\}\Big|_{\tau_1 = \cdots = \tau_j = 0},$$

we obtain

$$\begin{aligned} D^j F_1(u_0)(v_1, \ldots, v_j) &= (1-\partial_x^2)^{-1} \Big[\frac{k!}{(k-j)!} u_0^{k-j} (\partial_x^3 u_0) v_1 v_2 \cdots v_j \\ &+ \frac{k!}{(k-j+1)!} u_0^{k-j+1} \left((\partial_x^3 v_1) v_2 \cdots v_j + \cdots + v_1 \cdots v_{j-1} (\partial_x^3 v_j) \right) \Big], \end{aligned}$$

where $v_\ell \in G^{\delta, s+2}$, $j = 1, \ldots, k$. We also have that

$$D^{k+1} F_1(u_0)(v_1, \ldots, v_{k+1}) = (1-\partial_x^2)^{-1} \left[k! \left((\partial_x^3 v_1) v_2 \cdots v_{k+1} + \cdots + v_1 \cdots v_k (\partial_x^3 v_{k+1}) \right) \right],$$

and $D^j F_1(u_0) = 0$ for all $j > k+1$.

By using lemmas 2 and 3, for $0 < \delta' < \delta \leq 1$, $1 \leq j \leq k$ and $v_1, \ldots, v_j \in G^{\delta, s+2}$ and assuming that $s > 1/2$ we can estimate

$$\begin{aligned} \|D^j F_1(u_0)(v_1, \ \ldots \ , v_j)\|_{\delta', s+2} &\leq 2 \frac{k!}{(k-j)!} \|u_0^{k-j} (\partial_x^3 u_0) v_1 v_2 \cdots v_j\|_{\delta', s} \\ &+ 2 \frac{k!}{(k-j+1)!} \|u_0^{k-j+1} \left((\partial_x^3 v_1) v_2 \cdots v_j + \cdots + v_1 \cdots v_{j-1} (\partial_x^3 v_j) \right) \|_{\delta', s} \\ &\leq 2 c_s^k \frac{k!}{(k-j)!} \|u_0\|_{\delta', s}^{k-j} \|\partial_x^3 u_0\|_{\delta', s} \|v_1\|_{\delta', s} \|v_2\|_{\delta', s} \cdots \|v_j\|_{\delta', s} \\ &+ 2 c_s^k \frac{k!}{(k-j+1)!} \|u_0\|_{\delta', s}^{k-j+1} \|\partial_x^3 v_1\|_{\delta', s} \|v_2\|_{\delta', s} \cdots \|v_j\|_{\delta', s} \\ &+ \cdots + 2 c_s^k \frac{k!}{(k-j+1)!} \|u_0\|_{\delta', s}^{k-j+1} \|v_1\|_{\delta', s} \cdots \|v_{j-1}\|_{\delta', s} \|\partial_x^3 v_j\|_{\delta', s} \\ &\leq \frac{2 c_s^k e^{-1}}{\delta - \delta'} \frac{k!}{(k-j)!} \|u_0\|_{1, s+2}^{k-j+1} \|v_1\|_{\delta, s+2} \|v_2\|_{\delta, s+2} \cdots \|v_j\|_{\delta, s+2} \\ &+ \frac{2 c_s^k e^{-1}}{\delta - \delta'} \frac{k! j}{(k-j+1)!} \|u_0\|_{1, s+2}^{k-j+1} \|v_1\|_{\delta, s+2} \|v_2\|_{\delta, s+2} \cdots \|v_j\|_{\delta, s+2}. \end{aligned}$$

Notice now that

$$
\begin{aligned}
\frac{k!}{(k-j)!}+\frac{k!j}{(k-j+1)!} &= j!\left(\frac{k!}{j!(k-j)!}+\frac{k!}{(j-1)!(k-j+1)!}\right)\\
&= j!\left(\frac{k!}{j(j-1)!(k-j)!}+\frac{k!}{(j-1)!(k-j+1)(k-j)!}\right)\\
&= j!\left(\frac{k!}{(j-1)!(k-j)!}\left(\frac{1}{j}+\frac{1}{k-j+1}\right)\right)\\
&= j!\frac{k!}{(j-1)!(k-j)!}\frac{k+1}{j(k-j+1)}\\
&= j!\frac{(k+1)!}{j!(k-j+1)!}\\
&= j!\binom{k+1}{k-j+1}\\
&\leq j!2^{k+1},\ \forall\ 1\leq j\leq k.
\end{aligned}
$$

Hence, if we take $C_0=\dfrac{1}{\|u_0\|_{1,s+2}}$ and $A_1=c_s^k e^{-1}2^{k+2}\|u_0\|_{1,s+2}^{k+1}$ then we have that

$$\|D^jF_1(u_0)(v_1,\dots,v_j)\|_{\delta',s+2}\leq\frac{A_1C_0^j j!}{\delta-\delta'}\|v_1\|_{\delta,s+2}\|v_2\|_{\delta,s+2}\cdots\|v_j\|_{\delta,s+2}. \tag{4.4}$$

By proceeding analogously with the other two terms in (4.1), we have that

$$\|D^jF(u_0)(v_1,\dots,v_j)\|_{\delta',s+2}\leq\frac{AC_0^j j!}{\delta-\delta'}\|v_1\|_{\delta,s+2}\|v_2\|_{\delta,s+2}\cdots\|v_j\|_{\delta,s+2}, \tag{4.5}$$

where $A=(1+|b|)c_s^k 2^{k+3}e^{-1}\|u_0\|_{1,s+2}^{k+1}$.

Therefore, by Theorem 2 we conclude that the problem (1.1) has a unique solution, which for $0<\delta<1$ is a holomorphic function in the disc $D(0,T(1-\delta))$ valued in $G^{\delta,s+2}$. Moreover, the lifespan T is given by

$$T=\frac{1}{2e^2AC_0}=\frac{1}{c\|u_0\|_{1,s+2}^k}, \tag{4.6}$$

where $c=e(1+|b|)c_s^k 2^{k+4}$. The proof of Theorem 1 is now complete.□

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