Department of Mathematics, University of Aberdeen, Aberdeen, AB24 3UE, UK (m.crabb@abdn.ac.uk)
and
Departamento de Matemática, Universidade Federal de São Carlos, Caixa Postal 676, São Carlos, SP 13565-905, Brazil (pergher@dm.ufscar.br)

LIMITING CASES OF BOARDMAN'S FIVE HALVES THEOREM: CORRIGENDUM

MICHAEL C. CRABB AND PEDRO L. Q. PERGHER

Abstract. We rectify two omissions in the list of generators and include a brief discussion of the localization theorem of Kosniowski and Stong.

1. Corrections

We are grateful to J. M. Boardman (private communication, published as [1]) for pointing out two omissions in the list of generators given in [2].

The cases in which $c = 2$ in the main theorems 1.2 and 3.4 should be corrected to:

Theorem 1.2 [2, p. 724]

$$c = 2: \qquad 2 \ \textit{if } k = 1; \ 9 \ \textit{if } k = 2; \ 13 \ \textit{if } k = 3; \ 14 \ \textit{if } k \geqslant 4.$$

Theorem 3.4 [2, p. 730]

$$\begin{aligned}
c = 2: \quad & \textit{if } k \geqslant 1 && b^{k-1} \cdot x_3^{(2)},\ b^{k-1} \cdot \gamma(x_2^{(1)}),\\
& \textit{and, if } k \geqslant 2, && b^{k-2} \cdot (x_4^{(2)})^2,\ b^{k-2} \cdot (y_4^{(2)})^2,\ b^{k-2} \cdot \gamma(x_3^{(2)}) \cdot x_4^{(2)},\ b^{k-2} \cdot x_6^{(3)} \cdot x_2^{(1)},\\
& && b^{k-2} \cdot \gamma^3(x_5^{(4)}),\ b^{k-2} \cdot x_8^{(4)},\ b^{k-2} \cdot \gamma(x_7^{(4)}),\\
& \textit{and, if } k \geqslant 3, && b^{k-3} \cdot \gamma^2(x_7^{(4)}) \cdot x_4^{(2)},\ b^{k-3} \cdot \gamma^2(x_7^{(4)}) \cdot y_4^{(2)},\ b^{k-3} \cdot \gamma^2(x_{11}^{(6)}),\\
& && b^{k-3} \cdot x_2^{(1)} \cdot z_{11}^{(5)},\\
& \textit{and, if } k \geqslant 4, && b^{k-4} \cdot (\gamma^2(x_7^{(4)}))^2.
\end{aligned}$$

These require the following corrections to the text on page 729. To the list of exclusions when $c = 2$ must be added, if $k \geqslant 3$, $((6, 2_{k-3}), \omega_\emptyset))$. And, in the paragraph below the list, the dimension of the group $(\mathcal{N}_3^{\mathbb{Z}_2})^{(2)}$ should be corrected to $\dim (\mathcal{N}_3^{\mathbb{Z}_2})^{(2)} = 2$.

We note, also, that the exceptional case when $c = 3$ and $n = 2k - 1$ should read as $\omega = (2_{k-1})$, $\omega' = (1)$.

2. The localization theorem

We take this opportunity to place the result of Kosniowski and Stong [3] which provided the basic input into [2] in the context of what is now standard localization theory.

Cohomology with $\mathbb{Z}_2$-coefficients will be denoted by H^*. For a $\mathbb{Z}_2$-space M we write $H^*_{\mathbb{Z}_2}(M) = H^*(E\mathbb{Z}_2 \times_{\mathbb{Z}_2} M)$ for the equivariant Borel cohomology and let $t \in H^1_{\mathbb{Z}_2}(*)$ be the generator, that is, the Euler class of the universal real line bundle over $B\mathbb{Z}_2$. We have a restriction map $i^* : H^*_{\mathbb{Z}_2}(M) \to H^*(M)$. If $\mathbb{Z}_2$ acts trivially on M, then $H^*_{\mathbb{Z}_2}(M) = H^*(M) \otimes \mathbb{Z}_2[t]$.

Using the notation and terminology of [2] we can state the localization theorem for $\mathbb{Z}_2$-Borel cohomology as follows. Consider an m-dimensional $\mathbb{Z}_2$-manifold M with fixed-point data (F^j, η_j), $j = 0, \dots, m$. Suppose that $u \in H^m_{\mathbb{Z}_2}(M)$. Then

$$i^*(u)[M] = \sum_{j=0}^{m} (e(\eta_j)^{-1} u^{(j)})[F^j] \in \mathbb{Z}_2,$$

where $u^{(j)} \in H^m_{\mathbb{Z}_2}(F^j)$ is the restriction of u to F^j and $e(\eta_j) \in H^{m-j}_{\mathbb{Z}_2}(F^j)$ is the equivariant Euler class of η_j.

More explicitly, the equivariant Euler class $e(\eta_j)$ and its inverse can be written as

$$\begin{aligned} e(\eta_j) &= t^{m-j} + w_1(\eta_j)t^{m-j-1} + \cdots + w_{m-j}(\eta_j) \in H^*(F^j) \otimes \mathbb{Z}_2[t], \\ e(\eta_j)^{-1} &= t^{j-m}(1 + w_1(-\eta_j)t^{-1} + \cdots + w_j(-\eta_j)t^{-j}) \in H^*(F^j) \otimes \mathbb{Z}_2[t, t^{-1}]. \end{aligned}$$

The class $u^{(j)}$ may be expanded as $u^{(j)}_m + u^{(j)}_{m-1}t + \cdots + u^{(j)}_0 t^m$, where $u^{(j)}_i \in H^i(F^j)$, so that $(e(\eta_j)^{-1}u^{(j)})[F^j] = \sum_{i=0}^{j}(w_{j-i}(-\eta_j)u^{(j)}_i)[F^j] \in \mathbb{Z}_2$.

The result of Kosniowski and Stong [2, Proposition 2.5] is proved, when $f(X_1, \dots, X_m)$ is homogeneous of degree $d \leqslant m$, by taking $u = t^{m-d}v$, where v is obtained by substituting in $f(X_1, \dots, X_m)$ the rth Stiefel-Whitney class of $E\mathbb{Z}_2 \times_{\mathbb{Z}_2} TM$ for the rth elementary symmetric function in the X_i.

Proof. This may be proved by following the argument given by Atiyah and Segal in [**?**, Theorem 2.12] to establish the corresponding result for K-theory. □

References

ASM. F. Atiyah and G. B. SegalThe index of elliptic operators: II Ann. of Math.87(1968), 531–545J. M. Boardman, *Math. Rev.* (MR3109755, 2014). M. C. Crabb and P. L. Q. Pergher, Limiting cases of Boardman's five halves theorem, *Proc. Edinburgh Math. Soc.* **56**, (2013), 723–732. C. Kosniowski and R. E. Stong, Involutions and characteristic numbers, *Topology* **17**, (1978), 309–330.