(H,G)-COINCIDENCE THEOREMS FOR FREE G-SPACES

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ABSTRACT. Let us consider G a group acting freely on a Hausdorff paracompact topological space X and let Y be a k-dimensional metrizable space (or k-dimensional CW-complex). In this paper, by using the genus of X, $\mathrm{gen}\,(X, G)$, we prove (H, G)-coincidence theorems for maps $f : X \to Y$. Such theorems generalize the main theorem proved by Aarts, Fokkink and Vermeer in [1] and the main result proved by dos Santos and Coelho in [11].

Key words: (H, G)-coincidence point, free G-action, genus of a G-space.

1. Introduction

Suppose that X, Y are topological spaces, G is a group acting freely on X and $f : X \to Y$ is a continuous map. If H is a subgroup of G, then H acts on the right on each orbit Gx of G as follows: if $y \in Gx$ and $y = gx, g \in G$, then $hy = ghx$. Following [5, 6, 9], the concept of G-coincidence is generalized as follows: a point $x \in X$ is said to be a (H, G)-*coincidence point* of f if f sends every orbit of the action of H on the G-orbit of x to a single point. If H is the trivial subgroup, then every point of X is a (H, G)-coincidence. If $H = G$, this is the usual definition of coincidence. If $G = \mathbb{Z}_p$, with p prime, then a nontrivial (H, G)-coincidence point is a G-coincidence point.

Aarts, Fokkink and Vermeer [1, Theorem 1] proved that if $i : X \to X$ is a fixed-point free involution of a normal space X with color number $n+2$ and k is a natural number then for every k-dimensional cone CW-complex Y and every continuous map $\varphi : X \to Y$ there is a $\mathbb{Z}_2$-coincidence, whenever $n \geq 2k$; and this result is the best possible. Let us observe that for $X = S^n$ the result was obtained independently by Shchepin in [12]. Dos Santos and Coelho [11, Theorem 1.1], by using the genus of X, $\mathrm{gen}\,(X, \mathbb{Z}_p)$, generalized the Aarts, Fokkink and Vermeer's result for free $\mathbb{Z}_p$-actions, where p is prime.

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In this paper, we extended the results proved in [1, 11] for free G-actions, where G is a finite group. Specifically, we prove the following result:

Theorem 1.1. *Let G be a finite group which acts freely on a Hausdorff paracompact space X, with* $\text{gen}\,(X,G) \geq n+1$ *and let k be a natural number.*

(a) *If $n > |G|k$ and Y is a k-dimensional metrizable space, then every continuous map $f : X \to Y$ has a (H,G)-coincidence point, for some nontrivial subgroup $H \subset G$.*

(b) *If $n = |G|k$ and Y is a k-dimensional cone CW-complex, then every continuous map $f : X \to Y$ has a (H,G)-coincidence point, for some nontrivial subgroup $H \subset G$.*

(c) *If $n < |G|k$ and $gen\,(X,G) = n+1$, then there exists a k-dimensional cone CW-complex Y and a continuous map $f : X \to Y$ such that f has no G-coincidence points. In particular, if $G = H = \mathbb{Z}_p$, f has no (H,G)-coincidence points.*

Remark 1.2. Theorem 1.1 is a natural generalization of [11, Theorem 1.1]. In the particular case, where $G = H \cong \mathbb{Z}_p$, to detect (H,G)-coincidence points with H nontrivial subgroup of G is equivalent to detect G-coincidence points. Also, by using Theorem 2.3, which states that $\text{gen}(X,G) = n+1$ is equivalent to $\text{col}(X,G) = n+|G|$, we conclude that Theorem 1.1 generalizes [1, Theorem 1].

In the case that Y is a cone CW-complex, Theorem 1.1 shows that the inequality $n \geq |G|k$ is the best condition for the existence of (H,G)-coincidences. Moreover, for $n = |G|k$, this results can not be extended to a wider class of CW-complex Y of dimension k.

Example 1.3. It is enough to consider [11, Example 1.2], with $G = \mathbb{Z}_p$. Consider $Y = \Delta^{ps+p-2}_{s-1}$ (the $(s-1)$-skeleton of the $(ps+p-2)$-simplex) and $Y^* = \Pi_{i=1}^{p} Y^i - \Delta$, where $Y^i = Y$, for all i and Δ is the diagonal. We have that $G = \mathbb{Z}_p$ acts freely on Y^* and Y^* is a Hausdorff paracompact space. Moreover, it follows from [16] and [2] that $\text{gen}\,(Y^*, \mathbb{Z}_p) = p(s-1)+1$.

Define $\pi : Y^* \to Y$ by $\pi(y_1, \dots, y_p) = y_1$, for all $(y_1, \dots, y_p) \in Y^*$ and clearly π has no $\mathbb{Z}_p$-coincidence points. From this, we conclude that Theorem 1.1 does not hold in the case $n = |G|\,k$, when Y is any CW-complex.

2. Preliminaries

Aarts, Brouwer, Fokkink and Vermeer, in [2], defined the genus, $\text{gen}\,(X,G)$, in the sense of Švarc, as follows.

Let G be a finite group which acts freely on a Hausdorff paracompact space X. Let G^* denote $G\backslash\{e\}$. We say that an open subset U of X is a *color* if $U \cap g \cdot U = \emptyset$ for all $g \in G^*$ and we shall say that a cover $\mathcal{U}$ of X by colors is a *coloring*. If (X, G) admits a finite coloring, then the *color number* $\mathrm{col}\,(X, G)$ is the minimal cardinality of a coloring. If U is a color, then the set $G \cdot U = \bigcup_{g \in G} g \cdot U$ is called a *set of the first kind* and $G \cdot U$ is said to be *generated* by the color U. As G is a group, the collection $\{g \cdot U |\ g \in G\}$ is pairwise disjoint. The space X together with the group action is usually called a G-space.

Definition 2.1. Suppose that X is a G-space and let U be a color. The *genus*, $\mathrm{gen}\,(X, G)$, is defined as the minimal cardinality of a covering of X by sets of the first kind.

It follows from the definition that the genus in non-decreasing under equivariant maps.

Proposition 2.2. *Let X and Y be Hausdorff paracompact free G-spaces and let $f : X \to Y$ be a G-equivariant map. Then,* $\mathrm{gen}\,(X, G) \leq \mathrm{gen}\,(Y, G)$.

Hartskamp [8] and Bogatyi [3, Theorem 5] proved independently the following result:

Theorem 2.3. *Suppose that X is a Hausdorff paracompact G-space. The following statements are equivalent.*
(i) $\mathrm{gen}\,(X, G) = n + 1$;
(ii) $\mathrm{col}\,(X, G) = n + |G|$.

Other results in connection with Theorem 2.3 were proved in the papers of Steinlein [14, 15].
Krasnosel'skiĭ in [10], proved the following theorem:

Theorem 2.4. $\mathrm{gen}\,(S^n, \mathbb{Z}_p) = n + 1$.

For two topological spaces X and Y, we recall that the join $X * Y$ is the quotient space of $X \times Y \times I$ under the identifications $(x, y, 0) \sim (x, y', 0)$ and $(x, y, 1) \sim (x', y, 1)$, ie., the space of all lines segments joining points in X to points in Y. A nice way to write points of $X * Y$ is as formal convex combinations $[t_1 x + t_2 y]$, with $0 \leq t_i \leq 1$ and $t_1 + t_2 = 1$. If X and Y are G-spaces, then so is $X * Y$. With this identification the k-fold join $G * G * \cdots * G$ is constructed as the space of formal convex combinations $[t_1 g_1 + t_2 g_2 + \cdots + t_k g_k]$ with $g_i \in G$, $0 \leq t_i \leq 1$ and $t_1 + \cdots + t_k = 1$, $i = 1, \ldots, k$.

Definition 2.5. The k-fold join $G * G * \cdots * G$ with the standard action

$$g \cdot [t_1 g_1 + t_2 g_2 + \cdots + t_k g_k] = [t_1(g \cdot g_1) + t_2(g \cdot g_2) + \cdots + t_k(g \cdot g_k)],$$

is a free G-space, denoted by S^k_G.

From [2], it follows the following result:

Theorem 2.6. *Let X be a Hausdorff paracompact free G-space such that* $\operatorname{gen}(X,G) \leq k$. *Then, there exists a G-equivariant map $F : X \to S_G^k$.*

In [13], Švarc obtained the following theorem:

Theorem 2.7. *Suppose that X is a Hausdorff paracompact G-space of* $\dim X = n$. *Then,* $\operatorname{gen}(X,G) \leq n+1$.

In [2], Aarts, Brouwer, Fokkink and Vermeer proved that

Theorem 2.8. *Let G be a finite group.*
(i) $\operatorname{gen}(S_G^k, G) = k$.
(ii) *Suppose that X is a Hausdorff paracompact $(k-2)$-connected free G-space. Then, there exists a G-equivariant map $F : S_G^k \to X$. As a consequence,* $\operatorname{gen}(X,G) \geq k$.

3. Proof of Theorem 1.1

Proof. **Case (a)** $n > |G|k$**.** Suppose that $|G| = r$ and let $G = \{g_1, \ldots, g_r\}$ be a fixed enumeration of elements of G.

Consider the map $G \times Y^r \to Y^r$ given by

$$(g, (y_1, \ldots, y_r)) \mapsto (y_{\sigma_g(1)}, \ldots, y_{\sigma_g(r)}),$$

where the permutation σ_g is defined by $\sigma_g(i) = j$, and j is such that $g_i g = g_j$. This map is a left G-action on Y^r.

For a subgroup $H \subset G$, let $(Y^r)^H = \{z \in Y^r |\ h \cdot z = z,\ \forall\ h \in H\}$ be the fixed point set of H and $F = \bigcup_H (Y^r)^H$, where H runs over all nontrivial subgroups of G.

Let $Y^{**} = Y^r - F$. We have that Y^{**} is a metrizable free G-space.

If X is any space with a G-action, then a map $f : X \to Y$ induces an equivariant map $\phi : X \to Y^r$ given by

$$\phi(x) = (f(g_1 x), \ldots, f(g_r x)),\ \forall\ x \in X.$$

In fact,

$$\begin{aligned} \phi(gx) &= (f(g_1 g x), \ldots, f(g_r g x)) \\ &= (f(g_{\sigma_g(1)} x), \ldots, f(g_{\sigma_g(r)} x)). \end{aligned}$$

On the other hand, $g\phi(x) = g(f(g_1 x), \ldots, f(g_r x)) = g(y_1, \ldots, y_r)$, where $y_i = f(g_i x)$. Thus

$$\begin{aligned} g\phi(x) &= (y_{\sigma_g(1)}, \ldots, y_{\sigma_g(r)}) \\ &= (f(g_{\sigma_g(1)} x), \ldots, f(g_{\sigma_g(r)} x)) \\ &= \phi(gx). \end{aligned}$$

Suppose that $f : X \to Y$ has no (H, G)-coincidence points, for any nontrivial subgroup $H \subset G$. Then $\phi(X) \subset Y^{**}$ and $\phi : X \to Y^{**}$ is an equivariant map.

From Proposition 2.2, we conclude that $\operatorname{gen}(X, G) \leq \operatorname{gen}(Y^{**}, G)$.

By Theorem 2.7,

$$\operatorname{gen}(Y^{**}, G) \leq \dim Y^{**} + 1 \leq |G|k + 1. \tag{3.1}$$

Since $n > |G|k$, it follows that $\operatorname{gen}(X, G) \leq |G|k + 1 < n + 1$, which is a contradiction. This complete the proof of case **(a)**.

Case (b) $n = |G|\, k$. In this case, the strategy used is the following: we shall show that the upper bound of equation (3.1) can be reduced by one, i.e., we will prove that $\operatorname{gen}(Y^{**}, G) \leq |G|\, k$.

Let Y be a k-dimensional cone CW-complex, i.e., $Y = CA = \dfrac{A \times [0, 1]}{\sim}$, where A is a CW-complex of dimension $k - 1$ and $\sim$ is the following equivalence relation: $(a, 1) \sim (a', 1)$, for all $a, a' \in A$.

In this sense, we obtain coordinates for Y. A point in Y is represented by the class $[a,\, u]$, with $a \in A$ and $u \in [0, 1]$.

We take $Y^{**} = \prod_{i=1}^{r} Y^i - F$, where $Y^i = Y$, for all i, and $F = \bigcup_{H} (Y^r)^H$, where H runs over all nontrivial subgroups of G. We will denote by $\alpha : Y^{**} \to Y^{**}$ the map determined by the free G-action on Y^{**} given in the proof of the case (a). Using coordinates, α is defined by

$$\alpha([a_1, u_1], \ldots, [a_r, u_r]) = ([a_{\sigma_g(1)}, u_{\sigma_g(1)}], \ldots, [a_{\sigma_g(r)}, u_{\sigma_g(r)}]), \tag{3.2}$$

for all $a_1, \ldots, a_r \in A$ and for all $u_1, \ldots, u_r \in [0, 1]$, where the permutation σ_g is given by $\sigma_g(i) = j$, and j is such that $g_i g = g_j$.

Lemma 3.1. $\operatorname{gen}(Y^{**},\, G) \leq |G|\, k$.

Proof. Let $Z = [0, 1] \times \cdots \times [0, 1] \setminus \{(1, \ldots, 1)\}$ and let $s :\, Z \to Z$ be the map given by

$$s(u_1, \ldots, u_r) = (u_{\sigma_g(1)}, \ldots, u_{\sigma_g(r)}), \text{ for all } (u_1, \ldots, u_r) \in Z.$$

The projection $\pi :\, Y^{**} \to Z$ defined by

$$\pi([a_1, u_1], \ldots, [a_r, u_r]) = (u_1, \ldots, u_r), \text{ for all } ([a_1, u_1], \ldots, [a_r, u_r]) \in Y^{**},$$

is well-defined, it is continuous and it satisfies the condition: $s \circ \pi = \pi \circ \alpha$. In fact, for all $([a_1, u_1], \ldots, [a_r, u_r]) \in Y^{**}$, we have:

$$\begin{aligned} (s \circ \pi)([a_1, u_1], \ldots, [a_r, u_r]) &= s(u_1, \ldots, u_r) \\ &= (u_{\sigma_g(1)}, \ldots, u_{\sigma_g(r)}) \\ &= \pi([a_{\sigma_g(1)}, u_{\sigma_g(1)}], \ldots, [a_{\sigma_g(r)}, u_{\sigma_g(r)}]) \\ &= (\pi \circ \alpha)([a_1, u_1], \ldots, [a_r, u_r]). \end{aligned}$$

Now, let us consider the following subsets of Z:

$$\begin{aligned} W_1^0 &= \{2/3\} \times [2/3, 1] \times [2/3, 1] \times \cdots \times [2/3, 1]; \\ W_2^0 &= [2/3, 1] \times \{2/3\} \times [2/3, 1] \times \cdots \times [2/3, 1]; \\ &\vdots \\ W_r^0 &= [2/3, 1] \times [2/3, 1] \times \cdots \times [2/3, 1] \times \{2/3\}. \end{aligned}$$

$$\begin{aligned} W_1^1 &= \{1\} \times [0, 2/3] \times [0, 1] \times [0, 1] \times \cdots \times [0, 1]; \\ W_2^1 &= \{1\} \times [2/3, 1] \times [0, 2/3] \times [0, 1] \times \cdots \times [0, 1]; \\ &\vdots \\ W_{r-1}^1 &= \{1\} \times [2/3, 1] \times \cdots \times [2/3, 1] \times [0, 2/3]. \\ &\vdots \\ W_1^r &= [0, 2/3] \times [0, 1] \times [0, 1] \times \cdots \times [0, 1] \times \{1\}; \\ W_2^r &= [2/3, 1] \times [0, 2/3] \times [0, 1] \times \cdots \times [0, 1] \times \{1\}; \\ &\vdots \\ W_{r-1}^r &= [2/3, 1] \times [2/3, 1] \times \cdots \times [2/3, 1] \times [0, 2/3] \times \{1\}. \end{aligned}$$

Let us define

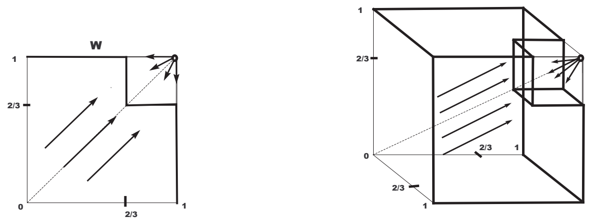
$$W = (\cup_{i=1}^r W_i^0) \cup (\cup_{j=1}^{r-1} W_j^1) \cup \ldots \cup (\cup_{j=1}^{r-1} W_j^r).$$

We have that W is the union of $r^2 = r + r(r-1)$ closed subsets of Z (Figure 1 illustrates the cases $r = 2$ and $r = 3$).

We define a retraction $\gamma : Z \to W$ as follows: in the right upper corner of Z, the retraction γ is the central projection to W with center of projection $(1, 1, \ldots, 1)$. In the lower part of Z, the retraction γ is the projection to W parallel to the diagonal of Z ($\{(z, z, \ldots, z) \mid z \in [0, 1]\}$).

Using the retraction γ, we define a retraction $\rho : Y^{**} \to \pi^{-1}(W)$ by

$$\rho([a_1, u_1], \ldots, [a_r, u_r]) = ([a_1, u_1'], \ldots, [a_r, u_r']),$$



(a) case $r = 2$ (b) case $r = 3$

FIGURE 1. The retraction $\gamma : Z \to W$.

for all $([a_1, u_1], \ldots, [a_r, u_r]) \in Y^{**}$, where $(u'_1, \ldots, u'_r) = \gamma(u_1, \ldots, u_r)$.

We have that ρ is well-defined and it is continuous. Moreover, it satisfies the condition $s \circ \gamma = \gamma \circ s$.

If we consider $\alpha' = \alpha|_{\pi^{-1}(W)}$, we have that $\alpha'(\pi^{-1}(W)) \subseteq \pi^{-1}(W)$ and, moreover, α' generates a free G-action on $\pi^{-1}(W)$.

Claim: $\rho : (Y^{**}, \alpha) \to (\pi^{-1}(W), \alpha')$ is a G-equivariant map. Indeed,

$$\begin{aligned} (\alpha' \circ \rho)([a_1, u_1], \ldots, [a_r, u_r]) &= \alpha'([a_1, u'_1], \ldots, [a_r, u'_r]) \\ &= ([a_{\sigma_g(1)}, u'_{\sigma_g(1)}], \ldots, [a_{\sigma_g(r)}, u'_{\sigma_g(r)}]), \end{aligned}$$

where $(u'_1, \ldots, u'_r) = \gamma(u_1, \ldots, u_r)$. On the other hand,

$$\begin{aligned} (\rho \circ \alpha)([a_1, u_1], \ldots, [a_r, u_r]) &= \rho([a_{\sigma_g(1)}, u_{\sigma_g(1)}], \ldots, [a_{\sigma_g(r)}, u_{\sigma_g(r)}]) \\ &= ([a_{\sigma_g(1)}, \tilde{u}_{\sigma_g(1)}], \ldots, [a_{\sigma_g(r)}, \tilde{u}_{\sigma_g(r)}]), \end{aligned}$$

where $(\tilde{u}_{\sigma_g(1)}, \ldots, \tilde{u}_{\sigma_g(r)}) = \gamma(u_{\sigma_g(1)}, \ldots, u_{\sigma_g(r)})$.

Note that, for some $l \in \{1, \ldots, r\}$, we have that $g^l = e$ (where e is the identity of G). Thus,

$$\begin{aligned} s^l(u_1, \ldots, u_r) &= s^{l-1}(u_{\sigma_g(1)}, \ldots, u_{\sigma_g(r)}) \\ &= s^{l-2}(u_{\sigma_{g^2}(1)}, \ldots, u_{\sigma_{g^2}(r)}) \\ &\vdots \\ &= s(u_{\sigma_{g^{l-1}}(1)}, \ldots, u_{\sigma_{g^{l-1}}(r)}) \\ &= (u_{\sigma_{g^l}(1)}, \ldots, u_{\sigma_{g^l}(r)}) \\ &\overset{\sigma_{g^l} = Id}{=} (u_1, \ldots, u_r). \end{aligned}$$

Now,

$$\begin{aligned} s^{l-1}(\tilde{u}_{\sigma_g(1)},\dots,\tilde{u}_{\sigma_g(r)}) &= s^{l-2}(\tilde{u}_{\sigma_{g^2}(1)},\dots,\tilde{u}_{\sigma_{g^2}(r)}) \\ &\vdots \\ &= (\tilde{u}_{\sigma_{g^l}(1)},\dots,\tilde{u}_{\sigma_{g^l}(r)}) \\ &\overset{\sigma_{g^l}=Id}{=} (\tilde{u}_1,\dots,\tilde{u}_r), \end{aligned}$$

and,

$$\begin{aligned} s^{l-1}(\gamma(u_{\sigma_g(1)},\dots,u_{\sigma_g(r)})) &\overset{s^{l-1}\circ\gamma=\gamma\circ s^{l-1}}{=} \gamma(s^{l-1}(u_{\sigma_g(1)},\dots,u_{\sigma_g(r)})) \\ &= \gamma(u_1,\dots,u_r). \end{aligned}$$

Therefore we have $(\tilde{u}_1,\dots,\tilde{u}_r) = \gamma(u_1,\dots,u_r) = (u'_1,\dots,u'_r)$. So, $s(\tilde{u}_1,\dots,\tilde{u}_r) = s(u'_1,\dots,u'_r)$ and we conclude that $(\tilde{u}_{\sigma_g(1)},\dots,\tilde{u}_{\sigma_g(r)}) = (u'_{\sigma_g(1)},\dots,u'_{\sigma_g(r)})$.

Consequently ρ is a G-equivariant map and thereby,

$$\mathrm{gen}(Y^{**},G) \leq \mathrm{gen}(\pi^{-1}(W),G).$$

It is easy to see that $\dim \pi^{-1}(W_i^j) \leq |G|k-1$, for all i and for all j. Since $\pi^{-1}(W) = \bigcup \pi^{-1}(W_i^j)$ is an union of closed subsets, thus by [4] (the sum theorem), it follows that $\dim \pi^{-1}(W) \leq |G|k-1$.

Thus, by using Theorem 2.7, we have

$$\mathrm{gen}(\pi^{-1}(W),G) \leq \dim \pi^{-1}(W) + 1 \leq (|G|k-1)+1 = |G|\,k.$$

Therefore,

$$\mathrm{gen}(Y^{**},G) \leq \mathrm{gen}(\pi^{-1}(W),G) \leq |G|\,k,$$

which completes the proof of lemma. □

Now, suppose that $f : X \to Y$ has no (H,G)-coincidence points. As in the proof of Theorem 1.1 (a), there is a G-equivariant map $\phi : X \to Y^{**}$. Then, it follows from Lemma 3.1, that

$$\mathrm{gen}(X,G) \leq \mathrm{gen}(Y^{**},G) \leq |G|\,k,$$

which contradicts $\mathrm{gen}(X,G) \geq n+1 = |G|k+1$. This completes the proof of Theorem 1.1 **(b)**.

Case (c) $n < |G|k$ **and** $\mathrm{gen}(X,G) = n+1$**.** In this case, $\mathrm{gen}(X,G) = n+1 \leq |G|\,k$, and it follows from Theorem 2.6 that there is a G-equivariant map $F : X \to S_G^{|G|k}$, where $S_G^{|G|k} = G * G * \cdots * G$ is the $|G|k$-fold join (Definition 2.5). On the other hand, by [16, Corollary 6.1] there exist a G-space X', a k-dimensional cone CW-complex Y and a continuous map $\psi_k : X' \to Y$ which has no G-coincidence points. Further, there is a G-equivariant map

$E : S_G^{|G|k} \to X'$, consequently, the map $f = \psi_k \circ E \circ F : X \to Y$ has no G-coincidence points. This completes the proof of case **(c)**. $\square$

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