On positive solutions for a class of singular quasilinear elliptic systems

O. H. Miyagaki*†
Departamento de Matemática, Universidade Federal de Viçosa
36571-000—Viçosa (MG), Brazil
e-mail: olimpio@ufv.br

R. S. Rodrigues‡
Departamento de Matemática, Universidade Federal de São Carlos
13565-905—São Carlos (SP), Brazil
e-mail: rodrigosrodrigues@ig.com.br & rodrigo@dm.ufscar.br

Abstract

We study through the lower and upper-solution method, the existence of positive weak solution to the quasilinear elliptic system with weights

$$\begin{cases} -div(|x|^{-ap}|\nabla u|^{p-2}\nabla u) &= \lambda|x|^{-(a+1)p+c_1}u^{\alpha}v^{\gamma} \text{ in } \Omega, \\ -div(|x|^{-bq}|\nabla v|^{q-2}\nabla v) &= \lambda|x|^{-(b+1)q+c_2}u^{\delta}v^{\beta} \text{ in } \Omega, \\ u = v &= 0 \qquad\qquad\qquad\quad \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded smooth domain of $\mathbb{R}^N$, with $0 \in \Omega$, $1 < p, q < N$, $0 \le a < \frac{N-p}{p}$, $0 \le b < \frac{N-q}{q}$, $0 \le \alpha < p-1$, $0 \le \beta < q-1$, δ, γ, c_1, $c_2 > 0$ and $\theta := (p-1-\alpha)(q-1-\beta) - \gamma\delta > 0$, for each $\lambda > 0$.

2000 Mathematical Subject Classifications: 35B05, 35D05, 35J70 and 35J40.
Key words and phrases: Degenerate equations, comparison theorems, strong maximum principle, positive solutions, quasilinear equations.

1 Introduction

In this paper, we will study through the lower and upper-solution method, the existence of positive weak solution to the quasilinear elliptic system with weights

$$\begin{cases} -div(|x|^{-ap}|\nabla u|^{p-2}\nabla u) &= \lambda|x|^{-(a+1)p+c_1}u^{\alpha}v^{\gamma} \text{ in } \Omega, \\ -div(|x|^{-bq}|\nabla v|^{q-2}\nabla v) &= \lambda|x|^{-(b+1)q+c_2}u^{\delta}v^{\beta} \text{ in } \Omega, \\ u = v &= 0 \qquad\qquad\qquad\quad \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

*Supported in part by CNPq-Brazil and AGIMB—Millenium Institute MCT/Brazil.
†Corresponding author.
‡Supported in part by Capes-Brazil