Regularizing effect of lower order terms in elliptic problems*

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Abstract

We analyze the regularizing effect due to the presence of a lower order terms ($h(x)|u|^{p-1}u$, $p \geq 1$) in several elliptic problems in a bounded open set Ω of $\mathbb{R}^N$.

First, for the case $p = 1$, we introduce some recent results obtained **jointly with Lucio Boccardo** (Università di Roma "Sapienza") about the regularizing effect of the interaction between the coefficient of the zero order term and the datum in some nonlinear Dirichlet problems. The simplest example is the linear problem

$$\begin{cases} -\mathrm{div}\,(M(x)\nabla u) + h(x)\,u = f(x), \text{ in } \Omega; \\ u = 0, \text{ on } \partial\Omega; \end{cases}$$

where M is a bounded elliptic matrix and $0 \leq h(x) \in L^1(\Omega)$. We show [1] that even if $f(x)$ only belongs to $L^1(\Omega)$, the assumption

$$\exists Q > 0 \text{ such that } |f(x)| \leq Q\,h(x), \text{ a.e. } x \in \Omega$$

implies the existence of a bounded (Hölder continuous) solution; i.e. a weak solution u belonging to $W_0^{1,2}(\Omega)$ and to $L^\infty(\Omega)$.

We also consider [2] the regularizing effect on the solutions when we have L^q interaction between the coefficient $h(x)$ and the datum $f(x)$ (i.e. Q is now a function in $L^q(\Omega)$).

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With respect to the case $p > 1$, we prove [4] (joint work with **Francisco Odair de Paiva** and **José M. Mendoza**, Universidade Federal de São Carlos, Brazil) that, contrarily with the case $h \equiv 0$, there exists a solution of the semilinear elliptic problem

$$\begin{cases} -\Delta u + h(x)|u|^{p-1}u = \lambda u + g(x,u) + f(x), & \text{in } \Omega \\ u = 0, & \text{on } \partial\Omega, \end{cases}$$

for every bounded Carathéodory function g in $\Omega \times \mathbb{R}$, $p > 1$, a nonnegative locally integrable function h in Ω which is strictly positive in a set of positive measure, $\lambda \in \mathbb{R}$ and $f \in L^2(\Omega)$.

We also consider (joint work [3] with **Alexis Molino** and **Lourdes Moreno** (Universidad de Granada) the case that $0 \in \Omega$ and a Hardy potential is introduced in the problem

$$\begin{cases} -\Delta u + h(x)|u|^{p-1}u = \lambda \dfrac{u}{|x|^2} + f(x) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

being in this case $\lambda > 0$ and $f \in L^m(\Omega)$ for some adequate $m > 1$.

References

[1] D. Arcoya, L. Boccardo: *Regularizing effect of the interplay between coefficients in some elliptic equations*; *Journal of Functional Analysis,* **268** (2015), 1053–1308.

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[3] D. Arcoya, A. Molino, L. Moreno-Mérida, *Existence and regularizing effect of degenerate lower order terms in elliptic equations beyond the Hardy constant*, preprint 2017.

[4] D. Arcoya, F. O. de Paiva, J. M. Mendoza, *Existence of solutions for a nonhomogeneous semilinear elliptic equation*, preprint 2017.