Barycenters of Polytope Skeleta via Constraints

Carlos H. F. Poncio

Abstract

The *Topological Tverberg Theorem* states that for any $d \geq 1$, prime power $r \geq 2$, and $N := (r-1)(d+1)$ every continuous map of the simplex Δ_N to $\mathbb{R}^d$ identifies points from r vertex-disjoint faces of Δ_N. This result was established for prime r by Bárány, Shlosman & Szűcs [2] in 1981, and for prime powers r in famous unpublished work by Özaydin [3] from 1987. The belief that the result should be equally valid for values $r \geq 6$ that are not prime powers, known as the *Topological Tverberg Conjecture*, was left open, as "a holy grail of topological combinatorics" (Kalai [4]).

The "constraint method," introduced in 2014 [5], shows that the Topological Tverberg Theorem implies virtually all subsequent extensions and sharpenings that were previously viewed as substantial independent results, such as the "Colored Tverberg Theorem" of Živaljević & Vrećica [6] or the "Generalized Van Kampen–Flores Theorem" of Sarkaria [7] and Volovikov [8].

In this talk, will be illustrated the mechanism and the power of the constraint method outside the classical "Tverberg type theorems," by a simple proof of a 2015 *Inventiones* result of Dobbins [9].

References

[1] Blagojević, P. V. M., Frick, F., Ziegler, G. M., Barycenters of Polytope Skeleta and Counterexamples to the Topological Tverberg Conjecture, via ConstraintsJ. Europ. Math. Soc., to appear (2018).

[2] Imre Bárány, Senya B. Shlosman, and András Szűcs, *On a topological generalization of a theorem of Tverberg*, J. Lond. Math. Soc. **23** (1981), 158–164.

[3] Murad Özaydin, *Equivariant maps for the symmetric group*, Preprint, 17 pages, http://digital.library.wisc.edu/1793/63829, 1987.

[4] Gil Kalai, *From Oberwolfach: The Topological Tverberg Conjecture is False*, "Combinatorics and more" blog post, February 6, 2015, https://gilkalai.wordpress.com/2015/02/06/from-oberwolfach-the-topological-tverberg-conjecture-is-false/gilkalai.wordpress.com.

[5] *Tverberg plus constraints*, Bull. Lond. Math. Soc. **46** (2014), no. 5, 953–967.

[6] Rade T. Živaljević and Siniša Vrećica, *The colored Tverberg's problem and complexes of injective functions*, J. Combin. Theory Ser. A **61** (1992), 309–318.

[7] Karanbir S. Sarkaria, *A generalized van Kampen–Flores theorem*, Proc. Amer. Math. Soc. **11** (1991), 559–565.

[8] Aleksei Yu. Volovikov, *On the van Kampen-Flores theorem*, Math. Notes **59** (1996), no. 5, 477–481.

[9] Michael G. Dobbins, *A point in a nd-polytope is the barycenter of n points in its d-faces*, Invent. Math. **199** (2015), 287–292.

(Carlos H. F. Poncio) UFSCar
e-mail: carlosponcio@dm.ufscar.br