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On the fractal dimension of global attractors

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Abstract

In this talk we discuss a result due to Mañé [3] that a compact subset $\mathcal{A}$ of a Banach space that is negatively invariant for a map S is finite-dimensional if $DS(x) = C(x) + L(x)$, where C is compact and L is a contraction (and both are linear). In particular, we see that if S is compact and differentiable then $\mathcal{A}$ is finite-dimensional. We also are interested in some results (following Málek et al. [2] and Zelik [4]) that give bounds on the fractal dimension of such sets assuming a 'smoothing property': in its simplest form this requires S to be Lipschitz from X into another Banach space Z that is compactly embedded in X. The resulting bounds depend on the Kolmogorov ε-entropy of the embedding of Z into X. We give applications to an abstract semilinear parabolic equation and the two-dimensional Navier–Stokes equations on a periodic domain.

It is a joint work with Alexandre N. Carvalho (USP), José Langa (Universidad de Sevilla - Spain) and James Robinson (University of Warwick - UK), see [1].

References

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