Regular and singular foliations on surfaces

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Abstract

The theory of foliations is a branch of geometry that originated in the early second half of the 20th century as the birth of the qualitative theory of differential equations, which related the theory of ordinary differential equations to differential topology. Precursor works in the theory of foliations belong to French mathematicians such as A. Haefliger, G. Ehresman, G. Reeb, among others. I. Nikolaev believes that G. Ehresman and G. Reeb were inspired by the works of H. Poincaré, A. M. Lyapunov, and G. D. Birkhoff, mathematicians who began using geometric methods in the study of differential equations. In fact, foliations naturally appear as solutions to systems of differential equations.

Intuitively, a foliation is a decomposition of a set into a union of disjoint, connected subsets of the same dimension. The main objective of this mini-course is to study some introductory results of the geometric theory of foliations. In particular, we will study the regular foliations in the plane given by submersions of the form $p(x, y) = r(x) + s(x)y$ where r and s are polynomial functions, and the singular foliations defined by Morse-Bott functions. In each case, we will see a complete topological invariant for its study.