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Fractional regularity for a doubly nonlinear differential inclusion

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Resumo

In this talk we are going to discuss some results concerning the questions on existence and regularity of solutions for the following doubly nonlinear differential inclusion:

$$\begin{aligned} \omega_t + \alpha(\omega_t) - \Delta\omega - \Delta_p\omega \ni f \quad &\text{in} \quad Q = \Omega \times (0,T), \\ \frac{\partial \omega}{\partial \nu} = 0 \quad &\text{on} \quad \partial\Omega \times (0,T), \\ \omega(.,0) = \omega_0 \quad &\text{in} \quad \Omega, \end{aligned} \tag{1}$$

where $\alpha \subset \mathbb{R}^2$ is a maximal monotone operator with $\alpha(0) \ni 0$ and Δ_p denotes the p-Laplacian with $p > 2$.

This problem goes back to solidification processes of certain polymers and has been investigated by several authors in the past years, and our interest in proving regularity results for (1) appeared while investigating such processes.

The main difficulty of problem (1) is concentrated in handling the interaction between the p-Laplacian and the nonlinearity $\alpha(\omega_t)$, what makes of (1) a doubly nonlinear differential inclusion. It is our aim to obtain the existence of "strong"solutions for our problem, and for this purpose we combine the so-called Rothe Method and regularity theory on sobolev spaces of fractional order.

Indeed, our technique is based on semi-discretization in time combined with regularity theory of an associate stationary problem and the Galerkin method. For this, we prove a fractional regularity up to the boundary and a result linking the eigenfunctions of the Laplacian with the Sobolev space $W^{1,p}(\Omega)$.

We intent to show the basic energy estimates and to discuss certain key lemmas used in the proof of the main result.