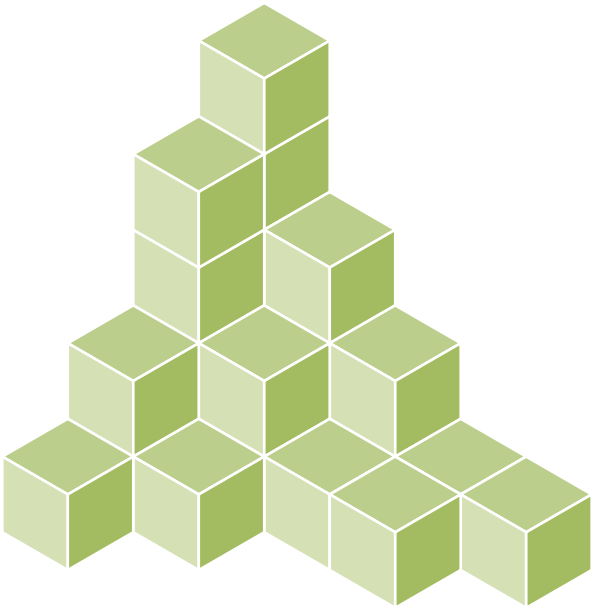
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Caderno de resumos
(Abstracts)

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PROGRAMAÇÃO (PROGRAM)

5ª feira.

13:30 - 14:00		Opening
14:00 - 14:45	Edivaldo L. dos Santos	*On the Bourgin–Yang theorem*
14:50 - 15:35	Marek Golasiński	*Generalized Gottlieb and Whitehead center groups of space forms*
15:40 - 16:05	Lucília D. Borsari	*Jordan Theorems for immersions in codimension one*
16:10 - 17:00		Poster Section I with Coffee
17:00 - 17:25	Alice Kimie Miwa Libardi	*Fixed Points on Non Trivial Surface Bundles*
17:30 - 17:50	Weslem Liberato Silva	*Periodic points of fiber-preserving maps on torus fiber bundles over the circle*
17:55 - 18:15	Juan Viu-Sos	*Combinatorics and topology of line arrangements via configurations of points*

6ª feira.

8:30 - 9:20	Michael R. Kelly	*Fixed points of homotopy idempotents*
9:25 - 9:50	Amanda Ferreira de Lima	*$\mathbb{Z}_2^k$-actions fixing $\mathbb{R}P^m \cup \mathbb{C}P^n$*
9:55 - 10:20	Oscar Ocampo	*$\mathbb{Z}_2-$manifolds arising from Artin braid groups*
10:20 - 10:40		Coffee
10:40 - 11:05	Leandro F. Aurichi	*Some topological games related to metrizable spaces*
11:10 - 12:00	Facundo Mémoli	*Persistent homology of asymmetric networks*
12:00 - 13:30		Lunch
14:00 - 14:45	Thaís F. M. Monis	*Fixed point property for Grassmannian manifolds*
14:50 - 15:35	Carolina de Miranda e Pereiro	*The Splitting of a Generalisation of the Fadell-Neuwirth short exact sequence*
15:40 - 16:05	Jamil V. Pereira	*Some applications of Topological Data Analysis in Biological Sciences*
16:10 - 17:00		Poster Section II with Coffee
17:00 - 17:25	Thiago de Melo	*On path-components of some mapping spaces*
17:30 - 17:50	Renato Vieira	*Recognition of relative loop spaces*
17:55 - 18:15	Julien Korinman	*Hyperbolic quantum links invariants from quantum Teichmüller theory and Volume conjectures*

Sábado.

8:30 - 9:20	Washington Mio	*What is a covariance tensor?*
9:25 - 9:50	Thaís Maria Dalbelo	*Brasselet number and Newton polygons*
9:55 - 10:20	Fabio Ferrari Ruffino	*Relative differential cohomology*
10:20 - 10:40		Coffee
10:40 - 11:05	Ana Maria Mathias Morita	*Free $\mathbb{Z}_2$-action on Dold manifolds and the cohomology ring of its orbit spaces*
11:10 - 12:00	Dirk Töben	*Riemannian Foliations and Equivariant Cohomology*
12:00 - 13:30		Lunch

PALESTRAS (TALKS)

Alice Kimie Miwa Libardi, D. L. Gonçalves, D. Penteado and J. P. Vieira
Fixed Points on Non Trivial Surface Bundles 7
Amanda Ferreira de Lima and Pedro Luiz Queiroz Pergher
$\mathbb{Z}_2^k$*-actions fixing* $\mathbb{R}P^m \cup \mathbb{C}P^n$ 9
Ana Maria Mathias Morita, Denise de Mattos and Pedro Luiz Queiroz Pergher
Free $\mathbb{Z}_2$*-action on Dold manifolds and the cohomology ring of its orbit spaces* 10
Carolina de Miranda e Pereiro
The Splitting of a Generalisation of the Fadell-Neuwirth short exact sequence 11
Oliver Goertsches and Dirk Töben
Riemannian Foliations and Equivariant Cohomology 12
Wacław Marzantowicz, Denise de Mattos and Edivaldo L. dos Santos
On the Bourgin–Yang theorem 13
Fabio Ferrari Ruffino and Juan Carlos Rocha Barriga
Relative differential cohomology 14
Facundo Mémoli
Persistent homology of asymmetric networks 15
L. Bordignon, T. de Melo, W. Mio, J.V. Pereira, C.J. Von Zuben
Some applications of Topological Data Analysis in Biological Sciences 16
Juan Viu-Sos and Benoît Guerville-Ballé
Combinatorics and topology of line arrangements via configurations of points 17
Julien Korinman
Hyperbolic quantum links invariants from quantum Teichmüller theory and Volume conjectures 18
Leandro F. Aurichi and Dione A. Lara
Some topological games related to metrizable spaces 19
Borsari, L.D. and Cardona, F.S.P. and Gonçalves, D.L.
Jordan Theorems for immersions in codimension one 20
Marek Golasiński and Thiago de Melo
Generalized Gottlieb and Whitehead center groups of space forms 21
Michael R. Kelly and Linda Hexter
Fixed points of homotopy idempotents 22
Oscar Ocampo
$\mathbb{Z}_2-$*manifolds arising from Artin braid groups* 23
Renato Vieira
Recognition of relative loop spaces 24
Thaís F. M. Monis and Peter Wong
Fixed point property for Grassmannian manifolds 25
Thaís Maria Dalbelo and Luiz Hartmann
Brasselet number and Newton polygons 26
Marek Golasiński, Thiago de Melo and Edivaldo L. dos Santos
On path-components of some mapping spaces 27
Washington Mio
What is a covariance tensor? 28
Weslem Liberato Silva and Rafael Moreira de Souza
Periodic points of fiber-preserving maps on torus fiber bundles over the circle 29

PAINÉIS I (POSTERS I)

Adriana Araujo Cintra and Irene Ignazia Onnis and Francesco Mercuri
Minimal surfaces in Lorentzian Heisenberg group and Damek-Ricci spaces via the Weierstrass representation 30
Anderson Paião dos Santos and Daciberg Lima Gonçalves
Free Diagonal Involutions on product of spaces 31
Apoenã Passos Passamani
Biconservative Surfaces in BCV-spaces 32
Bartira Maués
Considerations about the Wecken property of n-valued functions on surfaces 33
Cesar Augusto Ipanaque Zapata and Denise de Mattos
Collision free motion planning on manifolds with boundary 34
Daniel Vendrúscolo
Free involutions on some 3-manifolds 35
Letícia Sanches Silva and João Peres Vieira
Coincidence of maps on fiber bundles over the circle where the fiber is the torus 36
L. Hartmann
Inverse mapping theorem and local forms of continuous mappings 37
Maria Gorete Carreira Andrade and Ermínia de Lourdes Campello Fanti
Some contributions to the theory of Poincaré duality for groups and pairs 38
Pedro Benedini Riul
Comparing curvature loci of surfaces in euclidean spaces 39
Renato Monteiro de Moraes and Pedro Luiz Queiroz Pergher
Immersions of Dold and Wall manifolds in euclidean spaces 40
Thaís F. M. Monis and Northon C. L. Penteado and Sérgio T. Ura and Peter Wong
Nontrivial Intersection for Selfmaps of Complex Grassmann Manifolds 41
Denise de Mattos, Edivaldo L. dos Santos and Taciana O. Souza
(H, G)-coincidence theorems for manifolds and a topological Tverberg type theorem for any natural number r 42

PAINÉIS II (POSTERS II)

Adriano César Gois and Ermínia de Lourdes Campello Fanti
Relating ends of topological spaces and ends of groups 44
Alex Melges Barbosa
Euler Class 45
Bruno Caldeira Carlotti de Souza and Maria Gorete Carreira Andrade
An application of the non-relative Künneth Formula for topological joins 46
Guilherme Vituri F. Pinto
Topological Data Analysis 47
Larissa Moura and Alice Kimie Miwa Libardi
Spectral Clustering through Laplacian graphs and an application in soybean cultivation 48
Laurindo Daniel Silva da Rocha
Surface Branched Covers and Geometric 2-Orbifolds 49
Leandro Vicente Mauri and Denise de Mattos
Kneser's conjecture 50
Marco Antônio de Freitas Contessoto and Alice Kimie Miwa Libardi
Persistent homology to identify similar features in different animal foots 51
Mirela Cristina Marini, Ermínia de L. C. Fanti and Évelin M. Barbaresco
The Borsuk-Ulam Theorem: a weak version associated with topological groups 52
Rodrigo dos Santos Bononi and Ermínia de L. C. Fanti
An Introduction a splittings of Groups and some examples 53
Wagner Carvalho Sgobbi
Solution of the Conjugacy Problem for some extensions of groups 54

FIXED POINTS ON NON TRIVIAL SURFACE BUNDLES

ALICE KIMIE MIWA LIBARDI, D. L. GONÇALVES, D. PENTEADO, AND J. P. VIEIRA

Let $f : E \to E$ be a fibre-preserving map over S^1, i.e., $p \circ f = p$ where $p : E \to S^1$ is a fibre bundle with fibre a surface denoted by S (also called an S-bundle over S^1). When is f deformable over S^1 to a fixed point free map f' by a fibrewise homotopy over S^1? We remark that in order to have a positive answer a necessary condition is that the map f restricted to a fibre is deformable to a fixed point free map.

Now we consider surface bundles over S^1, $S \to E \to S^1$ where $E = \frac{S \times I}{(x,0) \sim (h(x),1)}$ for $h : S \to S$ a homeomorphism where E is not homeomorphic to the product $S \times S^1$. The case where h is the identity is known, see [GLPV13] and [GLPV].

A major question is: Given a homotopy class $\alpha \in [S, S]$ which has minimal number of fixed point μ, i.e., there is $g' \in \alpha$ which has μ fixed points and any other map on the homotopy class has at least μ fixed points, we would like to determine the homotopy classes of fibre maps $f : E \to E$ such that the restriction to the fibre is the class of α and over each fiber the map has exactly μ fixed points. In special the case where $\mu = 0$.

Let us make some comments which are relevant for problem, where we focus on the case where $\mu = 0$, i.e. the case when g can be deformed to a fixed point free map. The space E is the mapping torus of a homeomorphism of the surface S and the level of difficulty seems to us to be much higher than the case when the bundle is trivial. Due to the difficult of the problem een in this special case, one can break the problem into two problems.

More precisely, let us consider the map $\phi : [E, E]_{S^1} \to [S, S]$ which associates to a homotopy class of a fibre preserving map $[f]$ the homotopy class of the restriction $f|_S : S \to S$. Then one would like first to know which homotopy classes $[g] \in [S, S]$ which contain a fixed point free map are in the image of ϕ. This problem is related to some questions on the mapping class group of a surface. The answer is clear when the bundle is trivial (see [GLPV13]), but not in general. The other problem consist, for a given class $[g]$ which contains a function which is fixed point free and it is in the image of ϕ, we would like to compute $\phi^{-1}[g]$. Further among those homotopy classes in $\phi^{-1}[g]$, which ones contains a representative which is fixed point free fibrewise map.

In the case of a trivial bundle the questions above has been solved (see [GLPV]).

The problems above can be stated also for a surface bundle over a CW-complex B, and the solution for the case $B = S^1$ possibly is relevant to the understanding of the general case.

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$\mathbb{Z}_2^k$-ACTIONS FIXING $\mathbb{R}P^m \cup \mathbb{C}P^n$

AMANDA FERREIRA DE LIMA AND PEDRO LUIZ QUEIROZ PERGHER

A natural question in equivariant cobordism is the classification, up to equivariant cobordism, of smooth $\mathbb{Z}_2^k$-actions $(M;\phi)$, defined on closed and smooth manifolds M, with a given condition on the fixed-data of ϕ. The goal of this work is to present the equivariant cobordism classification of smooth $\mathbb{Z}_2^k$-actions for which the fixed point set of the action is the union $F = \mathbb{R}P^m \cup \mathbb{C}P^n$, when j and k are both even and greater than zero, $j \neq 2k$, obtained from the strong result proved in [2] and from the classification of the involutions fixing $F = \mathbb{R}P^m \cup \mathbb{C}P^n$, obtained in [1].

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FREE $\mathbb{Z}_2$-ACTION ON DOLD MANIFOLDS AND THE COHOMOLOGY RING OF ITS ORBIT SPACES

ANA MARIA MATHIAS MORITA, DENISE DE MATTOS AND PEDRO LUIZ QUEIROZ PERGHER

If G is a topological group and X is a topological space, a natural question asks for the existence of a continuous free action of G on X. If such an action exists, other natural question is the study of properties of the orbit space X/G and, in this setting, the study of the cohomology ring of X/G. This work is devoted to this question when X are special Dold manifolds and $G = \mathbb{Z}_2$.

The Dold manifolds $P(m, n)$ were introduced by A. Dold [3] with the purpose of finding odd-dimensional generators for the unoriented cobordism ring. Specifically, $P(m, n)$ is the orbit space of the free involution $-1 \times$ (conjugation) acting on $\mathbb{S}^m \times \mathbb{C}P^n$; that is, $P(m, n)$ is a closed smooth $(m + 2n)$-dimensional manifold. S. S. Khare [4] exhibited a fixed point free involution on $P(m, n)$ when n is odd. In this way, the study of the cohomology ring of $P(m, n)$ under this action makes sense when n is odd. The case $P(1, n)$ was proved recently by Morita, de Mattos and Pergher in [6].

The main tool in this context is the Leray-Serre spectral sequence associated to the Borel fibration coming from a G-action on X (see [1] and [5]).

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THE SPLITTING OF A GENERALISATION OF THE FADELL-NEUWIRTH SHORT EXACT SEQUENCE

CAROLINA DE MIRANDA E PEREIRO

The Fadell-Neuwirth short exact sequence for pure braid groups [1] can be generalised for the mixed braid group as follows:

$$1 \to B_m(M \setminus \{x_1, \ldots, x_n\}) \to B_{n,m}(M) \to B_n(M) \to 1, \tag{1}$$

And, more generally as:

$$1 \to B_{n_k}(M \setminus \{n_1 + \cdots + n_{k-1} \text{ pts}\}) \to B_{n_1,\ldots,n_k}(M) \to B_{n_1,\ldots,n_{k-1}}(M) \to 1 \tag{2}$$

We study the particular case where M is the torus or the Klein bottle and we give necessary and sufficient conditions for the splitting of the short exact sequence (1) and for the splitting of sequence (2) we have some partial results if M is the torus.

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RIEMANNIAN FOLIATIONS AND EQUIVARIANT COHOMOLOGY

OLIVER GOERTSCHES AND DIRK TÖBEN

The basic cohomology of a Riemannian foliation on a complete manifold with all leaves closed is the cohomology of the leaf space. In this paper [1] we introduce various methods to compute the basic cohomology in the presence of both closed and non-closed leaves in the simply-connected case (or more generally for Killing foliations): We show that the total basic Betti number of the union C of the closed leaves is smaller than or equal to the total basic Betti number of the foliated manifold, and we give sufficient conditions for equality. If there is a basic Morse-Bott function with critical set equal to C we can compute the basic cohomology explicitly. Our results are based on Molino's observation that the existence of non-closed leaves yields a distinguished transverse action on the foliated manifold with fixed point set C. We introduce equivariant basic cohomology of transverse actions in analogy to equivariant cohomology of Lie group actions enabling us to transfer many results from the theory of Lie group actions to Riemannian foliations. The prominent role of the fixed point set in the theory of torus actions explains the relevance of the set C in the basic setting. This technique was further applied in [2, 3, 4, 5].

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ON THE BOURGIN–YANG THEOREM

WACŁAW MARZANTOWICZ, DENISE DE MATTOS AND EDIVALDO L. DOS SANTOS

In 1933, S. Ulam posed and K. Borsuk showed that if $n > m$, then it is impossible to map $f : S^n \to S^m$ preserving symmetry in the sense that $f(-x) = -f(x)$. In 1954–55, C. T. Yang, and D. Bourgin, showed that if $f : S^n \to R^{m+1}$ preserves this kind of symmetry, then $\dim f^{-1}(0) \geq n - m - 1$.

We will present a survey of different versions of the latter for some other groups of symmetries, and also discuss the case $n = \infty$. Specifically, let V and W be orthogonal representations of a compact Lie group G with $V^G = W^G = \{0\}$. Let $S(V)$ be the sphere of V and $f : S(V) \to W$ be a G-equivariant mapping. We estimate the dimension of set $Z_f = f^{-1}\{0\}$ in terms of $\dim V$ and $\dim W$, where G is one of the following groups: the torus $\mathbb{T}^k$, the p-torus $\mathbb{Z}_p^k$, or the cyclic group $\mathbb{Z}_{p^k}$, where p is a prime. Also, we show that for any p-toral group $1 \to \mathbb{T}^k \to G \to \mathcal{P} \to 1$, $\mathcal{P}$ a finite p-group and a G-map $f : S(V) \to W$, with $\dim V = \infty$ and $\dim W < \infty$, we have that $\dim Z_f = \infty$.

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RELATIVE DIFFERENTIAL COHOMOLOGY

FABIO FERRARI RUFFINO AND JUAN CARLOS ROCHA BARRIGA

We provide a suitable axiomatic framework for differential cohomology in the relative case and we deduce the corresponding long exact sequences. We also construct the relative version of the generalized Cheeger-Simons characters and we define the integration map when the fibre has a boundary. Finally, we generalize the integration map to the relative cohomology of a morphism of fibre bundles.

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PERSISTENT HOMOLOGY OF ASYMMETRIC NETWORKS

FACUNDO MÉMOLI

I'll discuss recent work on trying to adapt persistent homology methods to datasets that exhibit asymmetry. Natural candidates are the Rips and Cech filtrations. Whereas the Rips filtration can unambiguously be directly generalized, generalizing the Cech filtration gives rise to two different versions: the sink and the source filtrations. It turns out that the Rips filtration imposes a symmetrization on the data whereas the Cech filtrations do not, thus making them more suitable for the analysis of intrinsically asymmetric data. By generalizing a theorem of Dowker we can prove that the persistent homologies of these two Cech filtrations are isomorphic. We establish the stability of these constructions under a metric between networks that generalizes the Gromov-Hausdorff distance. I'll also describe some results ve that characterize the persistence diagrams of some likely "motifs" in real (e.g. biological) networks: cycle-networks, which are directed analogues of the standard (discrete) circle.

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SOME APPLICATIONS OF TOPOLOGICAL DATA ANALYSIS IN BIOLOGICAL SCIENCES

L. BORDIGNON, T. DE MELO, W. MIO, J.V. PEREIRA, AND C.J. VON ZUBEN

We will discuss two basic examples of situations from biological sciences in which a computational approach using Topological Data Analysis (TDA) can be of great value, more precisely:

(1) We study phenotypic differences in soybean plants cultivated under four different conditions. We show how to transform leaf image data into functions that describe leaf contours and may be analyzed with TDA tools. The primary goal of the study is to investigate associations between variation in leaf morphology and environmental conditions.
(2) We study larvae of a fly species raised under several different conditions. Using pupal weight and size data, we quantify variation in developmental outcomes and analyze relationships with variation in environmental conditions.

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COMBINATORICS AND TOPOLOGY OF LINE ARRANGEMENTS VIA CONFIGURATIONS OF POINTS

JUAN VIU-SOS AND BENOÎT GUERVILLE-BALLÉ

A *line arrangement* $\mathcal{A} = \{L_0, \ldots, L_n\}$ is a finite collection of distinct lines in $\mathbb{CP}^2$, whose *combinatorics* (i.e. incidence relations between lines) is described by the associated *incidence graph* $\Gamma_{\mathcal{A}}$.

A classical problem of arrangements is the study of the influence of $\Gamma_{\mathcal{A}}$ on the topology of the embedding of $\mathcal{A}$, i.e. the homeomorphism type of the pair $(\mathbb{CP}^2, \mathcal{A})$. A couple of combinatorially equivalent line arrangements with different topology is called a *Zariski pair*.

At this time, there exist only three examples of Zariski pairs [6, 1, 4]), all of them defined over non-trivial number fields, and each of them distinguished by using computer calculations at some step. Among them, only one pair admits real equations, and it is the only case for which we do not know if it can be distinguished by the fundamental group of the complement.

Based on the linking $\mathcal{I}$-invariant introduced in [3], we present [5] a method to distinguish Zariski pairs admitting real equations, consisting on the weight counting of points of the dual configuration over particular chambers of $\mathbb{RP}^2$.

Using this point of view, we construct using GeoGebra a Zariski pair of 13 lines admitting equations over $\mathbb{Q}$, as well as other 9 new examples. Finally, we prove that the two line arrangements of this pair have non-isomorphic fundamental groups [2].

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HYPERBOLIC QUANTUM LINKS INVARIANTS FROM QUANTUM TEICHMÜLLER THEORY AND VOLUME CONJECTURES

JULIEN KORINMAN

The classification of knots and links is a deep outstanding problem in low dimensional topology. Huge progress have been made during the 80's thanks to Jone's discovery of new links invariants and Witten's interpretation in the context of Topological Quantum Field Theory. This first generation of quantum invariants is now well understood thanks to the works of Reshetikhin, Turaev, Rosso and others. Since then, two new generations of links invariants appeared, namely the ADO and quantum hyperbolic invariants whose physical interpretation remains mysterious. Kashaev conjectured that some of these new invariants should be related in the semi-classical limit to hyperbolic geometry, more precisely to the Gromov's norm of their exterior. In this talk, we will sketch a new construction of these links invariants related to quantum Teichmller theory and close to the geometry of the moduli space of SL_2-flat connexions and present Kashaev's conjecture and its variations.

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SOME TOPOLOGICAL GAMES RELATED TO METRIZABLE SPACES

LEANDRO F. AURICHI AND DIONE A. LARA

We present some topological games related to metrizable spaces. In particular, we present a game played over a metric space where the existence of a winning strategy for one of the players is equivalent to the existence of a equivalent metric over the space that is complete.

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JORDAN THEOREMS FOR IMMERSIONS IN CODIMENSION ONE

BORSARI, L.D., CARDONA, F.S.P., AND GONÇALVES, D.L.

In this work we intend to estimate the number of connected components of the complement $N - f(M)$, where $f\colon M \longrightarrow N$ is an immersion between closed, connected manifolds of dimension $(n-1)$ and n, respectively.

The papers [3], [1] and [2] studied that question and the paper by O. Saeki [4], generalizes several of the former results; the majority of those requires the manifolds to be orientable.

Using the local coefficient systems, we obtained a lower bound for the number of components of $N - f(M)$, that is the best possible, when the only multiple points are double points and the set of double points is a submanifold of M with dimension $(n-2)$.

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GENERALIZED GOTTLIEB AND WHITEHEAD CENTER GROUPS OF SPACE FORMS

MAREK GOLASIŃSKI AND THIAGO DE MELO

Given a map $f\colon A \to X$, in view of [1] (see also [3]), the m^{th} *generalized Gottlieb group* $G_m^f(X)$ is defined as the subgroup of the m^{th} homotopy group $\pi_m(X)$ consisting of all elements which can be represented by a map $\alpha\colon \mathbb{S}^m \to X$ such that $f \vee \alpha\colon A \vee \mathbb{S}^m \to X$ extends (up to homotopy) to a map $F\colon A \times \mathbb{S}^m \to X$. If $A = X$ then the group $G_1^f(X)$, also denoted by $J(f)$, is called the *Jiang group* of the map $f\colon X \to X$ in honor of Bo-Ju Jiang who recognized in [2] their importance to the Nielsen–Wecken theory of fixed point classes. The m^{th} *generalized Whitehead center group* $P_m^f(X)$, as defined in [3], is the set of all elements $\alpha \in \pi_m(X)$ whose Whitehead products $[\alpha, f\beta]$ are zero for all $\beta \in \pi_l(A)$ with $l \geq 1$. It turns out that $P_m^f(X)$ forms a subgroup of $\pi_m(X)$ and $G_m^f(X) \subseteq P_m^f(X)$.

Given a free action $H \times \mathbb{S}^{2n+1} \to \mathbb{S}^{2n+1}$ of a finite group H on $\mathbb{S}^{2n+1}$, Oprea [4] has shown that $G_1(\mathbb{S}^{2n+1}/H) = \mathcal{Z}H$, the center of H. The first main advantage that we take in this paper is to show that

$$G_1^f(\Sigma(2n+1)/H) = \mathcal{Z}_H f_*(\pi_1(A)),$$

the centralizer of $f_*(\pi_1(A))$ in H for a map $f\colon A \to \Sigma(2n+1)/H$, where $\Sigma(2n+1)$ is a $(2n+1)$-homotopy sphere. In particular, we get that the Jiang group $J(f) = \mathcal{Z}_H f_*(H)$ for any $f\colon \Sigma(2n+1)/H \to \Sigma(2n+1)/H$.

Further, we compute or estimate Gottlieb groups $G_m^f(\Sigma(2n+1)/H)$ and $P_m^f(\Sigma(2n+1)/H)$ for some $m > 1$, finite groups H and $f\colon A \to \Sigma(2n+1)/H$. In particular, we get some results on $G_m^f(L^{2n+1}(l; q_1, \ldots, q_n))$ for the $(2n+1)$-dimensional generalized lens space $L^{2n+1}(l; q_1, \ldots, q_n)$ and $f\colon A \to L^{2n+1}(l; q_1, \ldots, q_n)$.

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FIXED POINTS OF HOMOTOPY IDEMPOTENTS

MICHAEL R. KELLY AND LINDA HEXTER

The Bass Conjecture is a long standing problem concerning the algebraic K-theory of discrete groups. In the case that the group is finitely presented R. Geoghegan reformulated this conjecture as a problem in Nielsen fixed point theory. In this setting the spaces in question have π_1 as the given group, and the maps of interest are homotopy idempotent self maps on the space.

This talk represents an approach to this problem by an analysis of homotopy idempotents in the case of free groups. A characterization for all homotopy idempotents on the wedge of circles corresponding to the free group of rank 2 is given. By a direct study of Nielsen paths for these maps, one is then able to conclude that the Nielsen number is zero when the homotopy idempotent has rank one, and is equal to one otherwise. This provides a direct proof of the Bass Conjecture in this special case. A more general result, using methods from cyclic homology, was proved by B. Eckmann in 1986.

Results in the case of free groups of rank greater than two will also be discussed.

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$\mathbb{Z}_2-$MANIFOLDS ARISING FROM ARTIN BRAID GROUPS

OSCAR OCAMPO

Let B_n (resp. P_n) denote the Artin braid group (resp. pure Artin braid group) with n strings and let $\sigma\colon B_n \to S_n$ be the canonical projection. Suppose that $n \geq 3$. Consider a subgroup H of S_n isomorphic to $\mathbb{Z}_2$ and let $\widetilde{H} = \frac{\sigma^{-1}(H)}{[P_n,P_n]}$ be the Bieberbach subgroup of the crystallographic group $\frac{B_n}{[P_n,P_n]}$. Let $\mathcal{X}_H$ denote the $\mathbb{Z}_2-$manifold with fundamental group $\widetilde{H}$. In this work we will compute $Z(\widetilde{H})$, the center of $\widetilde{H}$, and taking quotients of $\widetilde{H}$ by subgroups of $Z(\widetilde{H})$ we obtain Bieberbach groups (resp. $\mathbb{Z}_2-$manifolds) of lower order than $\widetilde{H}$ (resp. $\mathcal{X}_H$). Also, we will decompose the holonomy representation of $\widetilde{H}$, $\psi_H\colon H \to Aut\left(\frac{P_n}{[P_n,P_n]}\right)$, in irreps and using this we will determine if the related $\mathbb{Z}_2-$manifold $\mathcal{X}_H$ admits Anosov diffeomorphism and (for even dimension) Kähler geometry.

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RECOGNITION OF RELATIVE LOOP SPACES

RENATO VIEIRA

In [1] May established (with later contribution by Cohen and Segal for the non-connected cases) a recognition principle for N-iterated loop spaces using the language of operads for any $1 \leq N \leq \infty$. A topological operad $\mathcal{O}$ is a set of spaces $\mathcal{O}(n)$ for each $n \in \mathbb{N}$ that can be thought of as a set of abstract multivariable operations, each one having n inputs and one output, which can be composed one with the others. It is a category-theoretic analog of universal algebra. An algebra over an operad is a space such that it admits operations parametrized by the operad. Specifically there is the little N-cubes operad $\mathcal{C}^N$ such that $\mathcal{C}^N(n)$ is the configuration space of n disjoint N-cubes embedded through translations and coordinate-wise dilation in I^N. It is simple to prove that every N-iterated loop space is a $\mathcal{C}^N$-algebra. For a pointed space Y, a CW-comples X and a weak equivalence $\tilde{\sigma} : X \to \Omega^N Y$ it is not generally true that X inherits a $\mathcal{C}^N$-algebra structure, but there is a model category structure on the category of topological operads, for which if $\overline{\mathcal{C}}^N$ is any cofibrant replacement of the little N-cubes operad (like a Boardman-Vogt resolution of $\mathcal{C}^N$ or the Fulton-MacPherson compactification of configuration spaces operad) then X has a $\overline{\mathcal{C}}^N$-algebra structure.

On the other hand using the two sided bar construction we can functorially build from a grouplike $\overline{\mathcal{C}}^N$-algebra X a $N-1$-connected space $B^N X$ (a connective Ω-spectrum when $N = \infty$) and a weak equivalence of pointed topological spaces $\tau : X \to \Omega^N B^N X$. $B^N X$ is known as the delooping of X. We also have that $B(\overline{\mathcal{C}}^N, \overline{\mathcal{C}}^N, X)$ is a cofibrant resolution of X in the model category of $\overline{\mathcal{C}}^N$-algebras and τ is adjoint to a weak equivalence of $\overline{\mathcal{C}}^N$-algebras $B(\overline{\mathcal{C}}^N, \overline{\mathcal{C}}^N, X) \to \Omega^N B^N X$. This recognition principle imply the existence of certain cohomology theories, and is an effective tool for the calculation of homology operations on X.

Using similar techniques we can demonstrate a recognition principle for relative loop spaces. There are generalizations of the notion of Operad, called colored operads, that parametrize operations with different possible outputs. In particular there is the N-Swiss Cheese operad $\mathcal{SC}^N$ whose algebras are pairs (X_o, X_c) such that X_c is a $\mathcal{C}^N$-algebra, X_o is a $\mathcal{C}^{N-1}$-algebra and X_c acts on X_o in a way compatible with the algebra structures. Again, there are cofibrant resolutions $\overline{\mathcal{SC}}^N$ and any pair of CW-complexes (X_o, X_c) weakly equivalent to a N-relative loop space pair $\Omega_2^N(A, Y) := (\Omega^N_{\text{rel}}(A, Y), \Omega^N Y)$ for some relative space (relative spectra for $N = \infty$) $A \subset Y$ is a $\overline{\mathcal{SC}}^N$-algebra. Not only that but we can also functorialy build $N-1$-connected relative spaces (connective relative spectra when $N = \infty$) $B_2^N(X_o, X_c)$ with properties analogous to May's delooping functor.

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FIXED POINT PROPERTY FOR GRASSMANNIAN MANIFOLDS

THAÍS F. M. MONIS AND PETER WONG

Given X, Y G-spaces, a G-map (or equivariant map) from X into Y is a map $f : X \to Y$ satisfying $f(g \cdot x) = g \cdot f(x)$, for all $g \in G$ and $x \in X$. Several results in topology are related with non-existence of equivariant maps. The most classical is the Borsuk-Ulam theorem.

In this talk, we are interested on the relationship between the fixed point property for spaces of the form G/K and the existence of K'-maps between certain spaces, where G is a compact Lie group, K is a closed subgroup of G, K' is a subgroup of K and G/K denotes the set of left cosets. A particular class of such spaces is the class of grassmannian manifolds.

The problem of determining the fixed point property for a grassmannian manifold has been studied by many authors (e.g. [1], [2], [3]). The main tool used until now is the calculation of the Lefschetz number of a self-map for such space. Since it is still an open problem, in this work we propose a different approach.

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BRASSELET NUMBER AND NEWTON POLYGONS

THAÍS MARIA DALBELO AND LUIZ HARTMANN

We present a formula to compute the Brasselet number of $f : (Y, 0) \to (\mathbb{C}, 0)$ where $Y \subset X$ is a non-degenerate complete intersection in a toric variety X. As applications we establish several results concerning about invariance of the Brasselet number for families of non-degenerate complete intersections. Moreover, when $(X, 0) = (\mathbb{C}^n, 0)$ we derive sufficient conditions to obtain the invariance of the Euler obstruction for families of complete intersections with isolated singularity which are contained on X.

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ON PATH-COMPONENTS OF SOME MAPPING SPACES

MAREK GOLASIŃSKI, THIAGO DE MELO, AND EDIVALDO L. DOS SANTOS

Let $M(X,Y)$ be the space of all continuous (not necessarily based) maps between connected spaces X and Y with the compact-open topology. A basic problem in homotopy theory is to determine whether two path-components of $M(X,Y)$ are homotopy equivalent or, more generally, to classify the path-components of $M(X,Y)$ up to homotopy type.

Works on these classification problems date back to the 1940's. Whitehead [7, Theorem 2.8] considered the case $X = \mathbb{S}^m$, the m-sphere and proved that $M_f(\mathbb{S}^m, Y)$ is homotopy equivalent to $M_0(\mathbb{S}^m, Y)$ if and only if the evaluation fibration $\omega_f : M_f(\mathbb{S}^m, Y) \to Y$ admits a section, where 0 denotes the constant map. Then, results by Hansen [2, 3], and later by McClendon [5] treated this classification problem as well.

Hansen [4] obtained a classification of path-components of $M(M^n, \mathbb{S}^n)$, where M^n is a suitably restricted n-manifold. The aim of [6] was to describe an approach which fits in well with Hansen's and leads to some further progress. In [1] the authors made use of Gottlieb groups of spheres to deal with path-components of the spaces $M(\mathbb{S}^{n+k}, \mathbb{S}^n)$ for $8 \leq k \leq 13$.

The aim of this work is to classify homotopy types of path-components of the space $M(E\Sigma^m, \mathbb{F}P^n)$ for certain m, n, where $E\Sigma^m$ is the reduced suspension of a homology m-sphere Σ^m.

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WHAT IS A COVARIANCE TENSOR?

WASHINGTON MIO

Covariance tensors are widely used to summarize the shape of Euclidean data. Extensions to Riemannian manifolds have been proposed, but they typically distort the data and do not properly account for the geometry of the underlying manifold. In this talk, I will discuss a different interpretation of classical covariance that only involves local linearity, as well as extensions to distributions on manifolds and graphs.

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PERIODIC POINTS OF FIBER-PRESERVING MAPS ON TORUS FIBER BUNDLES OVER THE CIRCLE

WESLEM LIBERATO SILVA AND RAFAEL MOREIRA DE SOUZA

Let M be a fiber bundle with base S^1 and fiber torus, $f : M \to M$ a fiber-preserving map over S^1 and $n > 1$ a positive integer. In this work we study the problem to find a map g fiberwise homotopic to f such that $g^n : M \to M$ is a fixed point free map over S^1, where g^n denotes the iterated n times of g. We present conditions that guarantee the existence of a such map g in some cases.

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MINIMAL SURFACES IN LORENTZIAN HEISENBERG GROUP AND DAMEK-RICCI SPACES VIA THE WEIERSTRASS REPRESENTATION

ADRIANA ARAUJO CINTRA, IRENE IGNAZIA ONNIS, AND FRANCESCO MERCURI

The Weierstrass representation formula is an important tool in the study of minimal surfaces in $\mathbb{R}^n$. The local version of the Weierstrass representation formula has been extended for minimal surfaces in a Riemannian manifold in [4] and in Lorentzian manifolds in [3]. However, the differential equations involved are quite difficult to handle and, in general, we need an ad hoc argument for each ambient space.

In this talk, we describe these arguments in the case of the 3-dimensional Heisenberg group and in the 4-dimensional Damek-Ricci group, equipped with a left invariant Lorentzian metric. For the case of spacelike surfaces we employ the complex analysis, and for timelike surfaces our approach makes use of the paracomplex analysis. Also, we use the Weierstrass type representation formula to construct interesting examples of spacelike and timelike minimal surfaces in these spaces.

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FREE DIAGONAL INVOLUTIONS ON PRODUCT OF SPACES

ANDERSON PAIÃO DOS SANTOS AND DACIBERG LIMA GONÇALVES

In this work we study a generalization of the Borsuk–Ulam theorem for maps from a product of two topological spaces, equipped with the diagonal involution, to the n-dimensional euclidian space. In the case of a product of two closed surfaces we give necessary and sufficient conditions for the validity of this generalization.

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BICONSERVATIVE SURFACES IN BCV-SPACES

APOENÃ PASSOS PASSAMANI

As suggested by Eells-Lemaire in [2], a *biharmonic map* $\phi : (M, g) \to (N, h)$ between Riemannian manifolds is a critical point of the bienergy functional:

$$E_2(\phi) = \frac{1}{2} \int_M |(\mathrm{d} + \mathrm{d}^*)^2 \phi|^2 \, \mathrm{dv_g} \ ,$$

where d is the exterior differential operator relative to the conection on $\phi^{-1}TM$ and d^* is the codifferential operator, the adjoint operator of d. This interesting class of maps includes the harmonic ones. As consequence, the set of the *biharmonic surfaces* includes the minimal ones and so, it has been extensively studied in the last two decades.

In [1], the authors introduce the concept of *biconservative hypersurfaces* as hypersurfaces with conservative *stress-energy* tensor with respect to the bienergy functional. The biconservative hypersurfaces form a geometrically interesting family which includes the biharmonic hypersurfaces.

In this work, we present the results obtained in [3] about the study of biconservative surfaces in the 3-dimensional Bianchi-Cartan-Vranceanu spaces, that can be thought as a local representation of simply connected homogeneous three-dimensional Riemannian manifolds. In particular, we obtain their characterization in the following cases: when they form a constant angle with the Hopf vector field; when they are SO(2)-invariant.

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CONSIDERATIONS ABOUT THE WECKEN PROPERTY OF N-VALUED FUNCTIONS ON SURFACES

BARTIRA MAUÉS

A central question in fixed point theory is if a space has the Wecken property, i.e. if it possible to deform any self-mapping f homotopically to a function with $N(f)$ fixed points, where $N(f)$ is the Nielsen number of f. This problem is already solved, and answered positively for closed, connected manifolds of dimension higher or equal than three and for the case of surfaces only the sphere, torus, the projective plane and the Klein bottle have the Wecken property.
The aim of the presentation is to give a first approach to the Wecken property of n-valued functions on surfaces. Similar as in the case for simple valued functions the Wecken property holds for compact triangulable manifolds of dimension at least three. The focus of this work is on the case of two-valued functions on the Klein-bottle and the question if it is possible to deform a function with Nielsen number zero homotopically to a function without fixed points. It is possible to state this problem in terms of braid theory and therefore, braid theory will be one of the important tools in this presentation.

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COLLISION FREE MOTION PLANNING ON MANIFOLDS WITH BOUNDARY

CESAR AUGUSTO IPANAQUE ZAPATA AND DENISE DE MATTOS - ORIENTADORA

In terms of Farber's topological complexity [1], we shows that the motion planning problem for a system consisting of non-colliding autonomous objects performing tasks in a manifold becomes topologically equivalent to the similar problem when the objects are restricted to do not touch the boundary in the process of motion.

This work is a part of my PhD's thesis under the supervision of professor Denise de Mattos and it is supported by FAPESP 2016/18714-8.

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FREE INVOLUTIONS ON SOME 3-MANIFOLDS: A GEOMETRIC VIEW

DANIEL VENDRÚSCOLO

We give explicity geometric constructions for some free involutions on certains class of 3-manifolds. In recent works the classification (up some relation) of free involutions on 3-manifolds have been studied as part of Borsu-Ulam type problems. We present geometric views for all classes of involutions on closed surfaces and on some families of 3-manifolds.

This is a work in progress with Claude Hayat.

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INVERSE MAPPING THEOREM AND LOCAL FORMS OF CONTINUOUS MAPPINGS

L. HARTMANN

We present a homological version of the Inverse Mapping Theorem for open and discrete continuous maps between oriented topological manifolds, with assumptions on the degree of the maps, but without any assumption on differentiability. We prove that this theorem is equivalent to the known homological version of the Implicit Mapping Theorem. Additionally, we study conditions for a map between oriented topological manifolds to be locally like an injection or a projection, via alternative notions of topological immersions and submersions. This is a joint work with A.P. Barreto (DM-UFSCar) and M.C. Fenille(UFU).

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COINCIDENCE OF MAPS ON FIBER BUNDLES OVER THE CIRCLE WHERE THE FIBER IS THE TORUS

LETÍCIA SANCHES SILVA AND JOÃO PERES VIEIRA

Let $f, g : M(\phi_1) \to M(\phi_2)$ be fiber preserving maps over the circle, S^1, where $M(\phi_1)$ and $M(\phi_2)$ are fiber bundles over S^1 and the fiber is the torus, T. The main purpose of this work is to classify the pairs of maps (f, g) which can be deformed by fiberwise homotopy over S^1 to a coincidence free pair (f', g'), $f', g' : M(\phi_1) \to M(\phi_2)$. This problem is related with an algebraic system of equations and it was motivated by problem addressed in [4], that consider this question in the case that $\phi_1 = \phi_2$.

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INVERSE MAPPING THEOREM AND LOCAL FORMS OF CONTINUOUS MAPPINGS

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We present a homological version of the Inverse Mapping Theorem for open and discrete continuous maps between oriented topological manifolds, with assumptions on the degree of the maps, but without any assumption on differentiability. We prove that this theorem is equivalent to the known homological version of the Implicit Mapping Theorem. Additionally, we study conditions for a map between oriented topological manifolds to be locally like an injection or a projection, via alternative notions of topological immersions and submersions. This is a joint work with A.P. Barreto (DM-UFSCar) and M.C. Fenille(UFU).

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SOME CONTRIBUTIONS TO THE THEORY OF POINCARÉ DUALITY FOR GROUPS AND PAIRS

MARIA GORETE CARREIRA ANDRADE AND ERMÍNIA DE LOURDES CAMPELLO FANTI

In this work we present some contributions to the theory of Poincaré duality for groups and pairs obtained, in the most part, through a family of invariant ends defined in [1] and characterized more elegantly in [4]. We also present some results about virtual Poincaré duality groups.

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COMPARING CURVATURE LOCI OF SURFACES IN EUCLIDEAN SPACES

PEDRO BENEDINI RIUL

At each point in an immersed surface in $\mathbb{R}^4$ there is a curvature ellipse in the normal plane which codifies all the local second order geometry of the surface ([2]). More recently, at the singular point of a corank 1 singular surface in $\mathbb{R}^3$, a curvature parabola in the normal plane which codifies all the local second order geometry has been defined ([3]). When projecting a regular surface in $\mathbb{R}^4$ to $\mathbb{R}^3$ in a tangent direction, corank 1 singularities appear generically. The projection has a cross-cap singularity unless the direction of projection is asymptotic, where more degenerate singularities can appear ([4]). In [1], we provide the relation between the geometry of an immersed surface in $\mathbb{R}^4$ at a certain point and the geometry of the projection of the surface to $\mathbb{R}^3$ at the singular point. In particular we relate the curvature ellipse of the surface to the curvature parabola of its singular projection.

The results are part of my PhD thesis supervised by M. A. S. Ruas and it is a joint work with R. Oset Sinha (Universitat de València, Spain).

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IMMERSIONS OF DOLD AND WALL MANIFOLDS IN EUCLIDEAN SPACES

RENATO MONTEIRO DE MORAES AND PEDRO LUIZ QUEIROZ PERGHER

Let M^n be a closed smooth n-dimensional manifold. Solving the immersion problem of M^n, means finding the number $i(M^n)$ such that M^n immerges in $\mathbb{R}^{i(M^n)}$ and does not immerge in $\mathbb{R}^{i(M^n)-1}$. We know, by Ralph Cohen's improvement of the famous Whitney's Theorem, that given an arbitrary closed smooth manifold M^n, it immerges in $\mathbb{R}^p$ with $p = 2n-\alpha(n)$, where $\alpha(n)$ is the number of ones in the binary expansion of n. It is known that any M^n does not immerge in $\mathbb{R}^n$, that is, in general $n+1 \leq i(M^n) \leq 2n - \alpha(n)$, even more, this estimate cannot be improved with such degree of generality.

In this work, we gave an original contribution in this direction, computing $i(M^n)$ for some Dold and Wall manifolds. This was done by calculation of the tangencial class' inverse and putting together such calculations with Cohen's Theorem.

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NONTRIVIAL INTERSECTION FOR SELFMAPS OF COMPLEX GRASSMANN MANIFOLDS

THAÍS F. M. MONIS, NORTHON C. L. PENTEADO, SÉRGIO T. URA, AND PETER WONG

Let $G(k,n)$ be the complex Grassmann manifold of k-planes in $\mathbb{C}^{k+n}$. The fixed point property (f.p.p.) for a Grassmann manifold $G(k,n)$ can be stated as follow: for any selfmap $f : G(k,n) \to G(k,n)$, there exists a k-plane $V^k \in G(k,n)$ such that $f(V^k) = V^k$. The problem of determining the f.p.p. for Grassmann manifolds has been studied by many authors (for example [5], [3], [4]). In this note, we prove a much more modest result for complex Grassmann manifolds than a fixed point theorem. Our main theorem is the following.

Let $k > 1$ and $k < n$. Then for every continuous map $f : G(k,n) \to G(k,n)$ there exists a k-plane $V^k \in G(k,n)$ such that $V^k \cap f(V^k) \neq \{0\}$.

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(H,G)-COINCIDENCE THEOREMS FOR MANIFOLDS AND A TOPOLOGICAL TVERBERG TYPE THEOREM FOR ANY NATURAL NUMBER r

DENISE DE MATTOS, EDIVALDO L. DOS SANTOS AND TACIANA O. SOUZA

Let G be a finite group which acts freely on a space X and let $f : X \to M$ be a continuous map from X into another space M. If H is a subgroup of G, then H acts on the right on each orbit Gx of G as follows: if $y \in Gx$ and $y = gx$, with $g \in G$, then $h \cdot y = gh^{-1}x$. A point $x \in X$ is said to be a (H,G)- *coincidence point of* f (as introduced by Gonçalves, Pergher and Jaworowski in [2]) if f sends every orbit of the action of H on the G-orbit of x to a single point. Let H be a cyclic subgroup of G of prime order p and let M be a connected m-manifold (orientable if $p > 2$) and $f^*(V_k) = 0$, for $k \geq 1$, where V_k are the Wu classes of M and suppose that $indX \geq n > (|G| - r)m$, where $r = \frac{|G|}{p}$. In this work, [1], we first estimate the cohomological dimension of the set $A(f, H, G)$ of (H,G)-coincidence points of f. Also we give similar estimative in the case that H is a p-torus subgroup of a particular group G and as application we prove a topological Tverberg type theorem for any natural number r.

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RELATING ENDS OF TOPOLOGICAL SPACES AND ENDS OF GROUPS

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The theory of ends of a group had its origin in the theory of ends of topological spaces. The number of ends of a topological space X, denoted by and $e(X)$, roughly saying, measures the "greatest amount"of "infinite"connected components (in other words, whose lock is non-compact) obtained when any compact is removed from X. In this work, we first present the definition of the number of ends of a group G, denoted by and $e(G)$, which is given in a totally algebraic form and some results. In the sequence, we present the definition of number of ends of a topological space, some examples and we approach the relation between ends of groups and of spaces, more precisely we explore, through examples, the following result: "If G is a finitely generated group, then the number of ends of G is equal to the number of ends of the space given by the Cayley Graph of G.

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EULER CLASS

ALEX MELGES BARBOSA

With the intention of conducting a brief study about the Euler class associated with an oriented bundle, we will start by saying when a vector bundle is an oriented bundle.

After that, we will define the Thom isomorphism associated with an oriented bundle and we will see some of its consequences.

We will start with some preliminary results that, according to the Euler class definition, were expected.

From these results, we will show some consequences with respect to the Euler class of a smooth and compact manifolds.

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AN APPLICATION OF THE NON-RELATIVE KÜNNETH FORMULA FOR TOPOLOGICAL JOINS

BRUNO CALDEIRA CARLOTTI DE SOUZA AND MARIA GORETE CARREIRA ANDRADE

The join operation is very important in the algebraic topology. For instance it is fundamental to Milnor's construction of the universal principal fibre bundle in [3] where he also prove the non-relative Künneth formula for joins. In this work we present an application of this formula in the theory of (H, G)-coincidences for maps. In more details, given a map $f : X \to Y$ whose domain is a free G-space, it is desirable to know a criterion which determines if it exists a point of (H, G)-coincidence for f in X, that is, a point $x \in X$ such that $f(H(O(x))) \subset \{f(x)\}$, where H is a non trivial subgroup of G. One of these criteria is presented in the paper [1] and, as application of the Künneth Formula for joins, we detail the example (contained in [1]) of a map $f : X \to Y$ whose domain is a free $\mathbb{Z}_p$-space, with p a prime integer, but does not have any $\mathbb{Z}_p$-coincidence points. This example shows that the inequality $m \geqslant |G|k$, which is one of the hypothesis required by the main theorem in [1], is the best possible condition that guaranties the existence of a coincidence point for certain types of maps.

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TOPOLOGICAL DATA ANALYSIS

GUILHERME VITURI F. PINTO

Given a finite metric space X, what can be said about its shape? Does it have "holes" or "voids"? In this work we will present a brief introduction about basic objects used in topological data analysis: the *Vietoris-Rips complex* of X and its induced filtration, which give rise (using the homology functor) to a sequence of vector spaces and linear transformations between them, called the *persistence module* of X; the resulting *barcode* and the *persistence diagram* of X, which summarize in an visual way the geometry of X; and finally the graph called *Mapper*, which extracts interesting information about the space X trought a given function $f : X \to \mathbb{R}$ and has some relation with its Reeb graph.

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SPECTRAL CLUSTERING THROUGH LAPLACIAN GRAPHS AND AN APPLICATION IN SOYBEAN CULTIVATION

LARISSA MOURA AND ALICE KIMIE MIWA LIBARDI (ADVISOR)

Clustering is a tool for exploring the structure of the data that does not require the assumptions common to most statistical methods. Clustering techniques play a very central role in data analysis with applications in areas such as biology, medicine and engineering. They can give important clues to the structure of databases and therefore can suggest results and hypotheses in the underlying sciences.

Cluster analysis is the formal study of algorithms and methods for grouping or classifying objects.

In recent years, methods based on graph Laplacians have become increasingly popular, in the literature there are three matrix that are knowing as Laplacian graph.

In this work we will study clustering through Laplacian graphs and we will present an application in the study of soybean cultivation. We will use the unnormalized Laplacian spectral clustering and their eigenvectors, and present the steps of a clustering algorithm.

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SURFACE BRANCHED COVERS AND GEOMETRIC 2-ORBIFOLDS

LAURINDO DANIEL SILVA DA ROCHA

The purpose of this work is to study the realizability problem of branched coverings between closed, connected and orientable surfaces. For each covering, there exists a set of naturally associated data called *branch datum* that should satisfy the Riemann-Hurwitz formula.

A classical problem (for possibly non-orientable surfaces) asks whether for a branch datum satisfying the condition of Riemann-Hurwitz exists a branched cover between surfaces having it as branch datum. The correct answer is: not always. When a branch datum satisfies the necessary conditions to come from a branched covering, we call it a *candidate branched covers*; if indeed it comes from a branched cover we call it *realizable* and, if not, we call it *exceptional*.

In fact, it is known that exceptions can occur only if the covered surface is the sphere or the projective plane, but the general solution is still unknown.

Among the various tools used to attack the problem we will work directly with the *geometric 2-orbifolds*.

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KNESER'S CONJECTURE

LEANDRO VICENTE MAURI AND DENISE DE MATTOS - ORIENTADORA

In this work we will prove the Kneser's Conjecture (an interesting problem of Graph Theory) using topological methods, more specifically the Borsuk-Ulam Theorem.

Kneser's Conjecture: For all $k > 0$ and $n \geq 2k - 1$, the chromatic number of the Kneser graph $KG_{n,k}$ is $n - 2k + 2$.

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PERSISTENT HOMOLOGY TO IDENTIFY SIMILAR FEATURES IN DIFFERENT ANIMAL FOOTS

MARCO ANTÔNIO DE FREITAS CONTESSOTO AND ALICE KIMIE MIWA LIBARDI - ORIENTADORA

Persistent homology is a method for identify topological patterns and features in a set of data. We, first, transform this set into a set of point in $\mathbb{R}^n$ and, with this set, construct a family of simplicial complexes that will be used to compute homology, generating a barcode, which probably gives us information about topological features in our set.
Sometimes we have different sets that give us barcodes that are really similar and, for this, is important use functional persistent, that is, to use a function $f : X \subset \mathbb{R}^n \longrightarrow \mathbb{R}$, where X is our set in $\mathbb{R}^n$, to have a filtration of our set and study the homology in each filtration to see if the topological features are the same in all of them.
In this work, we are going to compute the persistent homology in some animal foots, analyse this to see big differences that split some of them and if we have similar barcodes, we are going to try to see a difference between them, using functions as density and height.

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THE BORSUK-ULAM THEOREM: A WEAK VERSION ASSOCIATED WITH TOPOLOGICAL GROUPS

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The Borsuk-Ulam Theorem classical states that: "If $f : S^n \to \mathbb{R}^n$ is any continuous map, then there exists a point x in S^n such that $f(x) = f(-x)$, or equivalently $f(x) = f(A(x))$, where S^n denotes the n-dimensional unit sphere and $A : S^n \to S^n$ is the antipodal map". This work is based on the article "Some generalizations of Borsuk-Ulam Theorem" by Vendrúsculo, Desideri and Pergher (2011), [2], and has the main purpose of presenting a study of a weak version of the Borsuk-Ulam Theorem associated with topological groups given by the authors. It is said that $\{(X,T);G\}$, where X is a topological space equipped with a free involution T and G is a topological group "satisfies a Weak version of the Borsuk-Ulam Theorem", abbreviatedly, "satisfies WBUT" if, given any continuous map $f : X \to Y$, the set $\{x \in X;\ f(x) \cdot f(T(x))^{-1} \in 2G\}$ is different from empty, where $f(T(x))^{-1}$ is the symmetric of $f(T(x))$ in G and $2G = \{g \in G;\ g = g^{-1}\}$. In this work, we relate this weak condition with the more general condition of "satisfying the Borsuk-Ulam Theorem" (or "satisfying BUT"); we give some examples; considering $G = \mathbb{T}^2$ (torus), we present the proof of a result that establishes an algebraic criterion for $\{(X,T);\mathbb{T}^2\}$ satisfy the condition WBUT, and of a result that gives an equivalence between the weak version WBUT for triples $\{(\mathbb{S},T);\mathbb{T}^2\}$ and the condition BUT for $\{(\mathbb{S},T);\mathbb{R}^2\}$, where $\mathbb{S}$ is a closed surface and T is a free involution on $\mathbb{S}$ (using a theorem by Gonçalves (2006), [1]). Finally, we present a topological invariant obtained from the WBUT version. Such invariant, defined by us, is similar to that obtained from the BUT condition and presented by the cited authors.

References

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AN INTRODUCTION A SPLITTINGS OF GROUPS AND SOME EXAMPLES

RODRIGO DOS SANTOS BONONI AND ERMÍNIA DE LOURDES CAMPELLO FANTI

Let G a group and S a subgroup of G. We say that G splits over S if G is a non-trivial free with S amalgamated, so that $G = G_1 *_S G_2$, with $G_1 \neq S \neq G_2$ or G is a HNN-group with associated subgroup S, so that $G = G_1 *_{S,\theta}$ with θ some monomorphism from S to G_1. If G splits over some subgroup, we say that G is splittable. The amalgamation property of groups has many applications in several areas of the pure and applied mathematics such as graph theory, compact surfaces theory, ends of groups, logic, among others. In 1949 the mathematicians G. Higman, B.H. Neumann and H. Neumann proved several famous embedding theorems for groups using a construction later called the HNN-extension or HNN-group. The theory of HNN-groups is central to geometric and combinatorial group theory and can be viewed in parallel with amalgamated free products. In this work we define amalgamated free product and HHN-group, we present an introduction to the study of splittings of groups with some examples, especially when G is the fundamental groups of an orientable compact surface.

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SOLUTION OF THE CONJUGACY PROBLEM FOR SOME EXTENSIONS OF GROUPS

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In this work we studied the solution of the Conjugacy Problem for several classes of group extensions, obtained by O. Bogopolski, A. Martino, O. Maslakova and E. Ventura. Among the results, we point out its solution for free-by-cyclic groups and for some free extensions of the form $\mathbb{Z}^n \rtimes_A \mathbb{Z}$, or $\mathbb{Z}^2 \rtimes_{A_1,...,A_m} F_m$, or $F_2 \rtimes_{\varphi_1,...,\varphi_m} F_m$ (with $A \in GL_n(\mathbb{Z})$, $A_1, ..., A_m \in GL_2(\mathbb{Z})$ and $\varphi_1, ..., \varphi_m \in Aut(F_2)$), as well as $\mathbb{Z}^n \rtimes_{A_1,...,A_m} F_m$ with $\langle A_1, ..., A_m \rangle \leq GL_n(\mathbb{Z})$ a finite index or virtually solvable subgroup, or, finally, for groups of the type $F_n \rtimes_{\varphi_1,...,\varphi_m} F_m$, with $\langle \varphi_1, ..., \varphi_m \rangle \leq Aut(F_n)$ a finite index subgroup. We also studied solutions of the Twisted Conjugacy Problem for finitely generated free, polycyclic and surface groups.

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