083119 - Métodos de Matemática Aplicada

Terceira lista de exercícios

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1. Calcule $\mathcal{L}\{f(t)\}$.

   (a) $f(t) = t^n e^{at}$ (b) $f(t) = te^{at}\,\text{sen}\,(bt)$ (c) $f(t) = te^{at}\cos(bt)$ (d) $f(t) = t\,\mu_2(t)$
   (e) $f(t) = \cos(2t)\mu_\pi(t)$ (f) $f(t) = (t-1)^3 e^{t-1}\mu_1(t)$ (g) $f(t) = t^2\cos t$ (h) $f(t) = e^{2-t}\mu_2(t)$

2. Calcule $\mathcal{L}^{-1}\{F(s)\}$.

   (a) $F(s) = \dfrac{1}{(s-1)^4}$ (b) $F(s) = \dfrac{e^{-2s}}{s^3}$ (c) $F(s) = \dfrac{(1+e^{-2s})^2}{s+2}$ (d) $F(s) = \dfrac{se^{-\frac{\pi s}{2}}}{s^2+4}$
   (e) $\dfrac{e^{-s}}{s(s+1)}$ (f) $F(s) = \dfrac{s}{(s^2+1)^2}$

3. Resolva os PVIs.

   a) $\begin{cases} y' + 4y = e^{-4t} \\ y(0) = 2 \end{cases}$ b) $\begin{cases} y'' - 4y' + 4y = t^3 e^{2t} \\ y(0) = 0 \\ y'(0) = 0 \end{cases}$ c) $\begin{cases} y'' - y' = e^t\cos(t) \\ y(0) = 0 \\ y'(0) = 0 \end{cases}$

   d) $\begin{cases} y^{(4)} - y = 0 \\ y(0) = 1,\ y'(0) = 0,\ y''(0) = -1,\ y'''(0) = 0 \end{cases}$

   e) $\begin{cases} y'' + 4y = \text{sen}\,(t)\mathcal{U}(t-2\pi) \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$

   f) $\begin{cases} y'' + y = f(t) \\ y(0) = 0 \\ y'(0) = 1 \end{cases}$ onde $f(t) = \begin{cases} 0 & 0 \le t < \pi \\ 1 & \pi \le t < 2\pi \\ 0 & t \ge 2\pi \end{cases}$

4. Prove a comutatividade, a distributividade e a associatividade para a convolução.

   (a) $f * g = g * f$
   (b) $f * (g + h) = f * g + f * h$
   (c) $f * (g * h) = (f * g) * h$

5. Calcule $\mathcal{L}\{f(t)\}$ onde

   (a) $f(t) = \displaystyle\int_0^t (t-\tau)^2\cos 2\tau d\tau$ (b) $f(t) = \displaystyle\int_0^t e^{-(t-\tau)}\,\text{sen}\,\tau d\tau$ (c) $f(t) = t\displaystyle\int_0^t \text{sen}\,\tau d\tau$

   (d) $f(t) = \displaystyle\int_0^t (t-\tau)e^\tau d\tau$ (e) $f(t) = \displaystyle\int_0^t \text{sen}\,(t-\tau)\cos\tau d\tau$ (f) $f(t) = 1 * t^3$

6. Usando convolução calcule a transformada inversa das seguintes funções:

   (a) $\dfrac{1}{(s-4)(s-3)}$ (b) $\dfrac{s}{(s^2+1)(s-3)}$

   (c) $\dfrac{1}{s^2-2s+1}$ (d) $\dfrac{1}{s^2(s-5)}$

7. Resolva as seguintes equações integrais.

   (a) $y(t) = 5t + \displaystyle\int_0^t y(\tau)\,\text{sen}\,(t-\tau)d\tau$
   (b) $y(t) = 2\,\text{sen}\,4t + 3\displaystyle\int_0^t y(\tau)\,\text{sen}\,4(t-\tau)$