ON CYLINDRICALLY BOUNDED $H$-HYPERSURFACES OF $\mathbb{H}^n \times \mathbb{R}$

GREGORIO PACELLI BESSA

Abstract. Calabi, [1] in the sixties asked whether there were complete bounded minimal surfaces in $\mathbb{R}^3$. It is well known that this question was completely answered. See [5], [3] for non-existence of complete bounded minimal surfaces with bounded sectional curvature, see [2] for non-existence of complete bounded minimal surfaces with sectional curvature with strong quadratic decay and see [4] for the first example of a complete bounded minimal surface in $\mathbb{R}^3$. Naturally, one can ask whether there are complete bounded minimal surfaces in $\mathbb{H}^2 \times \mathbb{R}$. We prove in a joint work with Silvana Costa the following theorem

Theorem 1. Let $M$ be a complete hypersurface immersed in $\mathbb{H}^n \times \mathbb{R}$ with Ricci curvature with strong quadratic decay. If $M$ is cylindrically bounded then $\sup_M \geq (n - 1)/n$.

PS:

Definition 2. A complete Riemannian manifold $M$ has Ricci curvature $\text{Ric}_M$ with strong quadratic decay if

$$\text{Ric}_M(x) \geq -c^2 \left[ 1 + \rho_M^2(x) \log^2(\rho_M(x) + 2) \right],$$

where $\rho_M$ is the distance function on $M$ to a fixed point $x_0$ and $c = c(x_0) > 0$ is a constant depending on $x_0$.

References


Universidade Federal do Ceará - Brazil
E-mail address: bessa@mat.ufc.br