Errata of the book *Intermediate Spectral Theory and Quantum Dynamics, Progress in Mathematical Physics 54, Birkhäuser, Basel, 2008, by César R. de Oliveira.*

Corrections appear in chronological order:

- page 15, line -10. The left hand side of the polarization identity should read: $\langle \eta, \xi \rangle$

- page 143. The top of page should read:

  2. For bounded operators A, B and all $n \in \mathbb{N}$, one has (expand the r.h.s.)

  $$A^n - B^n = \sum_{j=0}^{n-1} A^j \, (A - B) \, B^{n-1-j},$$

  and the choices $A = e^{-itT/n} e^{-itS/n}$, $B = e^{-it(T+S)/n}$ imply that, for any $n \in \mathbb{N}$,

  $$\left(e^{-itT/n} e^{-itS/n}\right)^n - \left(e^{-it(T+S)/n}\right)^n = \cdots$$

- page 150, line +1. The form b_1 should be supposed to be closed in the KLMN Theorem.

- page 248, lines +3 and +4. It should be $S = S_1 + iS_2$ with $S_2 = \frac{i}{2}(S^* - S)$.

- page 227, line +10. The central part of the equation should read

  $$\mathbf{1} - \frac{1}{2} \left(W_t(x) + W_t(x)^2\right)$$

- page 264, line +10. The set Λ_ε should read $\Lambda_\varepsilon = (a + \varepsilon, b - \varepsilon)$.

- page 265, line +17. It should be $\|f_\varepsilon(T_n)\| < 1$ instead of $\|f_\varepsilon\|_\infty < 1$.

- page 348, last equation in the proof of Proposition 12.6.1. It should be

$$Y = \bigcap_{j,n\in\mathbb{N}} \bigcup_{t\in\mathbb{N}} \Big\{ T \in X : \langle p^T_{\psi_j} \rangle(t) < \frac{1}{n} \Big\}.$$

(I thank Jonás Arista, from Mexico, for this correction.)

- page 173. The proof of Theorem 7.1.13 needs to be corrected, since it was implicitly assumed that rng $\rho_1|_{\text{dom } T_{\hat{\mathcal{U}}}}$ is also dense in $\mathbf{h}$. In fact, the very definition of boundary triples (Definition 7.1.11) should include the assumption that the map

$$\begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} : \text{dom } T^* \to \mathbf{h} \times \mathbf{h}$$

is onto (instead of the ranges of both ρ_1 and ρ_2 be dense in $\mathbf{h}$). The proof of Theorem 7.1.13 then goes as follows.

Given a unitary $\hat{\mathcal{U}}$, as in the original proof $T_{\hat{\mathcal{U}}}$ is hermitian and it will be argued that $T^*_{\hat{\mathcal{U}}} \subset T_{\hat{\mathcal{U}}}$, so that $T_{\hat{\mathcal{U}}}$ is self-adjoint. Let $\eta \in \text{dom } T^*_{\hat{\mathcal{U}}}$. The relation $0 = \langle \rho_1(\eta), \rho_1(\xi) \rangle - \langle \rho_2(\eta), \rho_2(\xi) \rangle$, for all $\xi \in \text{dom } T_{\hat{\mathcal{U}}}$ (see page 174), implies the orthogonality

$$0 = \left\langle \begin{pmatrix} \rho_1(\xi) \\ \rho_2(\xi) \end{pmatrix}, \begin{pmatrix} \rho_1(\eta) \\ -\rho_2(\eta) \end{pmatrix} \right\rangle,$$

and since the graph $\mathcal{G}(\hat{\mathcal{U}})$ is a (closed) subspace of $\mathbf{h} \times \mathbf{h}$, the set

$$\left\{ \begin{pmatrix} \rho_1(\xi) \\ \rho_2(\xi) \end{pmatrix} : \xi \in \text{dom } T_{\hat{\mathcal{U}}} \right\} = \left\{ \begin{pmatrix} \rho_1(\xi) \\ \rho_2(\xi) \end{pmatrix} : \xi \in \text{dom } T^*, \rho_2(\xi) = \hat{\mathcal{U}}\rho_1(\xi) \right\}$$

coincides with the graph of $\hat{\mathcal{U}}$. By Lemma 2.1.15 and the above orthogonality, $\begin{pmatrix} \rho_2(\eta) \\ \rho_1(\eta) \end{pmatrix} \in \mathcal{G}(\hat{\mathcal{U}}^*)$, that is, $\hat{\mathcal{U}}^*\rho_2(\eta) = \rho_1(\eta)$, and so $\rho_2(\eta) = \hat{\mathcal{U}}\rho_1(\eta)$. Therefore, $\eta \in \text{dom } T_{\hat{\mathcal{U}}}$.

This modification of the definition of boundary triple also affects some arguments on page 198, since the ranges of both $\rho_1(\psi)$ and $\rho_2(\psi)$ are not

the whole space $\mathbf{h} = \mathrm{L}^2(S)$. So the conclusion that H_U is self-adjoint holds true under the following additional assumptions: (i) $-1 \in \rho(U)$ and (ii) if $\psi \in \mathcal{H}^1(S)$, then $(\mathbf{1}+U)^{-1}(\mathbf{1}-U)\psi \in \mathcal{H}^{1/2}(S)$ (see Theorem 4.5 of J. Berhndt, M. Langer, V. Lotoreichik, Spectral estimates for resolvent differences of self-adjoint elliptic operators. Preprint: arXiv:1012.4596).

Remark: In the particular case $U = e^{iu(\varphi)}$ (see item 4 on page 199), to guarantee that H_U is self-adjoint it is sufficient to impose that (j) $-1 \notin \operatorname{essrng} e^{iu(\varphi)}$ and (jj) $\nabla e^{iu(\varphi)} \in \mathrm{L}^\infty(S)$, which ensure (i) and (ii) above, respectively.

(I warmly thank Jussi Behrndt and Vladimir Lotoreichik, from Austria, for sharing their expertise with me and this important correction.)

- page 320. Include the hypothesis that the eigenfunction ψ_λ in Proposition 12.2.9 is of class $C^1(\mathcal{T})$. Explanation: the conditions $\varphi = |\psi_\lambda|^2 \in \mathrm{L}^1(\mathcal{T})$ and differentiable almost everywhere (as ψ_λ was supposed to be in $\mathcal{H}^1(\mathcal{T})$) does not seem to be enough to guarantee the convergence of the Fourier series of φ and its derivatives (at least I'm not aware of such result). Surely C^1 is too strong, but safe.

- page 347, line 14. Besides $\|(\mathbf{1}-P_E)TP_E\| < 2a/n$ it is also true that $\|P_ET(\mathbf{1}-P_E)\| < 2a/n$, since the latter operator is the adjoint of the former (as remarked in the text). Conclude then that (it should be line 15)

$$\|S\| < 4a/n,$$

and apply Lemma 12.5.4 directly to S, instead of $(\mathbf{1}-P_E)TP_E$ and $P_ET(\mathbf{1}-P_E)$, since S is self-adjoint, as required in the lemma.

- page 343, Proposition 12.4.12 is flawed. The term in line 5 does not vanish as $\lambda \to \infty$; the corresponding remarks should be ignored.

- page 26. The statement of Proposition 1.3.29 should be: Let $S_n, S \in \mathrm{B}(\mathcal{H})$ with $S_n \xrightarrow{\mathrm{s}} S$. If T is a compact operator, then $TS_n^* \to TS^*$ and $S_nT \to ST$ in the norm of $\mathrm{B}(\mathcal{H})$.

The second conclusion is detailed in the book; the proof of the first one may be obtained by the relation $\|T^*(S_n^* - S^*)\| = \|(S_n - S)T\|$ and

taking into account that T^* is also compact and so one may replace T with T^* in the original argument. Alternatively, one may reproduce the proof in the book directly for TS_n^*. This may be adapted to the proof of Theorem 11.3.4, which is the only application of Proposition 1.3.9 in the book.

(I thank Esteban Cardenas for pointing out this correction.)