
Corrections ordered by page numbers

- page 15, line -10. The left hand side of the polarization identity should read: \( \langle \eta, \xi \rangle \)

- page 26. The statement of Proposition 1.3.29 should be: Let \( S_n, S \in \text{B}(\mathcal{H}) \) with \( S_n \xrightarrow{\text{s}} S \). If \( T \) is a compact operator, then \( TS_n^* \rightarrow TS^* \) and \( S_n T \rightarrow ST \) in the norm of \( \text{B}(\mathcal{H}) \).

The second conclusion is detailed in the book; the proof of the first one may be obtained by the relation
\[
\| T^* (S_n^* - S^*) \| = \|(S_n - S)T\|
\]
and taking into account that \( T^* \) is also compact and so one may replace \( T \) with \( T^* \) in the original argument. Alternatively, one may reproduce the proof in the book directly for \( TS_n^* \). This may be adapted to the proof of Theorem 11.3.4, which is the only application of Proposition 1.3.9 in the book.

(I thank Esteban Cardenas for pointing out this correction.)

- page 143. The top of page should read:

2. For bounded operators \( A, B \) and all \( n \in \mathbb{N} \), one has (expand the r.h.s.)
\[
A^n - B^n = \sum_{j=0}^{n-1} A^j (A - B) B^{n-1-j},
\]
and the choices \( A = e^{-itT/n} e^{-itS/n}, B = e^{-it(T+S)/n} \) imply that, for any \( n \in \mathbb{N}, \)
\[
(e^{-itT/n} e^{-itS/n})^n - (e^{-it(T+S)/n})^n = \ldots
\]

- page 150, line +1. The form \( b_t \) should be supposed to be closed in the KLMN Theorem.
The proof of Theorem 7.1.13 needs to be corrected, since it was implicitly assumed that $\text{rng } \rho_1|_{\text{dom } T_{\hat{U}}}$ is also dense in $\mathfrak{h}$. In fact, the very definition of boundary triples (Definition 7.1.11) should include the assumption that the map

$$\begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} : \text{dom } T^* \to \mathfrak{h} \times \mathfrak{h}$$

is onto (instead of the ranges of both $\rho_1$ and $\rho_2$ be dense in $\mathfrak{h}$). The proof of Theorem 7.1.13 then goes as follows.

Given a unitary $\hat{U}$, as in the original proof $T_{\hat{U}}$ is hermitian and it will be argued that $T_{\hat{U}}^* \subset T_{\hat{U}}$, so that $T_{\hat{U}}$ is self-adjoint. Let $\eta \in \text{dom } T_{\hat{U}}^*$. The relation $0 = \langle \rho_1(\eta), \rho_1(\xi) \rangle - \langle \rho_2(\eta), \rho_2(\xi) \rangle$, for all $\xi \in \text{dom } T_{\hat{U}}^*$ (see page 174), implies the orthogonality

$$0 = \left\langle \begin{pmatrix} \rho_1(\xi) \\ \rho_2(\xi) \end{pmatrix}, \begin{pmatrix} \rho_1(\eta) \\ -\rho_2(\eta) \end{pmatrix} \right\rangle,$$

and since the graph $\mathcal{G}(\hat{U})$ is a (closed) subspace of $\mathfrak{h} \times \mathfrak{h}$, the set

$$\left\{ \begin{pmatrix} \rho_1(\xi) \\ \rho_2(\xi) \end{pmatrix} : \xi \in \text{dom } T_{\hat{U}} \right\} = \left\{ \begin{pmatrix} \rho_1(\xi) \\ \rho_2(\xi) \end{pmatrix} : \xi \in \text{dom } T^*, \rho_2(\xi) = \hat{U}\rho_1(\xi) \right\}$$

coincides with the graph of $\hat{U}$. By Lemma 2.1.15 and the above orthogonality, $\begin{pmatrix} \rho_2(\eta) \\ \rho_1(\eta) \end{pmatrix} \in \mathcal{G}(\hat{U}^*)$, that is, $\hat{U}^*\rho_2(\eta) = \rho_1(\eta)$, and so $\rho_2(\eta) = \hat{U}\rho_1(\eta)$. Therefore, $\eta \in \text{dom } T_{\hat{U}}$.

This modification of the definition of boundary triple also affects some arguments on page 198, since the ranges of both $\rho_1(\psi)$ and $\rho_2(\psi)$ are not the whole space $\mathfrak{h} = L^2(S)$. So the conclusion that $H_U$ is self-adjoint holds true under the following additional assumptions: (i) $-1 \in \rho(U)$ and (ii) if $\psi \in \mathcal{H}^1(S)$, then $(1 + U)^{-1}(1 - U)\psi \in \mathcal{H}^{1/2}(S)$ (see Theorem 4.5 of J. Berhndt, M. Langer, V. Lotoreichik, Spectral estimates for resolvent differences of self-adjoint elliptic operators. Preprint: arXiv:1012.4596).

Remark: In the particular case $U = e^{iu(\varphi)}$ (see item 4 on page 199), to guarantee that $H_U$ is self-adjoint it is sufficient to impose that (j) $-1 \notin$
essrng $e^{iu(\varphi)}$ and (jj) $\nabla e^{iu(\varphi)} \in L^\infty(S)$, which ensure (i) and (ii) above, respectively.

(I warmly thank Jussi Behrndt and Vladimir Lotoreichik, from Austria, for sharing their expertise with me and this important correction.)

- page 227, line +10. The central part of the equation should read
  $$1 - \frac{1}{2} (W_t(x) + W_t(x)^2)$$

- page 248, lines +3 and +4. It should be $S = S_1 + iS_2$ with $S_2 = \frac{i}{2} (S^* - S)$.

- page 264, line +10. The set $\Lambda_\varepsilon$ should read $\Lambda_\varepsilon = (a + \varepsilon, b - \varepsilon)$.

- page 265, line +17. It should be $\|f_\varepsilon(T_n)\| < 1$ instead of $\|f_\varepsilon\|_\infty < 1$.

- page 320. Include the hypothesis that the eigenfunction $\psi_\lambda$ in Proposition 12.2.9 is of class $C^1(\mathcal{T})$. Explanation: the conditions $\varphi = |\psi_\lambda|^2 \in L^1(\mathcal{T})$ and differentiable almost everywhere (as $\psi_\lambda$ was supposed to be in $\mathcal{H}^1(\mathcal{T})$) does not seem to be enough to guarantee the convergence of the Fourier series of $\varphi$ and its derivatives (at least I’m not aware of such result). Surely $C^1$ is too strong, but safe.

- page 320. Include the hypothesis that the eigenfunction $\psi_\lambda$ in Proposition 12.2.9 is of class $C^1(\mathcal{T})$. Explanation: the conditions $\varphi = |\psi_\lambda|^2 \in L^1(\mathcal{T})$ and differentiable almost everywhere (as $\psi_\lambda$ was supposed to be in $\mathcal{H}^1(\mathcal{T})$) does not seem to be enough to guarantee the convergence of the Fourier series of $\varphi$ and its derivatives (at least I’m not aware of such result). Surely $C^1$ is too strong, but safe.

- page 347, line 14. Besides $\|(1 - P_E)TP_E\| < 2a/n$ it is also true that $\|P_ET(1 - P_E)\| < 2a/n$, since the latter operator is the adjoint of the former (as remarked in the text). Conclude then that (it should be line 15)
  $$\|S\| < 4a/n,$$
  and apply Lemma 12.5.4 directly to $S$, instead of $(1 - P_E)TP_E$ and $P_ET(1 - P_E)$, since $S$ is self-adjoint, as required in the lemma.
• page 348, last equation in the proof of Proposition 12.6.1. It should be

\[ Y = \bigcap_{j,n \in \mathbb{N}} \bigcup_{t \in \mathbb{N}} \left\{ T \in X : \langle p^T_{\psi_j} \rangle (t) < \frac{1}{n} \right\}. \]

(I thank Jonás Arista, from Mexico, for this correction)